TRANSPORT
2008
23(2): 135–137

A MATHEMATICAL MODEL OF TRAIN CONTINUOUS MOTION UPHILL

Gintautas Bureika

Dept of Railway Transport, Vilnius Gediminas Technical University, J. Basanavičiaus g. 28, LT-03224 Vilnius, Lithuania, e-mail: gintautas.bureika@ti.vgtu.lt

Received 10 April 2007; accepted 1 February 2008

Abstract. The determination problem of adhesion coefficient of locomotives' wheels with rails is described in this paper. The use of methods of mathematical statistics and theory of probability provides wider possibilities for determining the most acceptable (suitable) value of the adhesion coefficient. The random factors influencing on this value are analysed. The use of the adhesion coefficient values based on reliable research results could help maintain uninterrupted (continuous) traffic of heavy freight trains and increase not only the carrying capacity, but also the volume of the transported goods. The paper considers a mathematical model of steady traffic of heavy freight trains pulled by upgraded locomotives of the series 2M62M on railway line with a gradient. The research results show that the actual values of the adhesion coefficient and the total resistance are distributed according to the normal distribution law. A mode of locomotive motion largely depends on the relationship between the total train's running resistance and locomotive tractive force. Finally, basic conclusions are given.

Keywords: heavy freight trains, track line gradient, locomotive tractive force, train continuous motion, coefficient of adhesion, running resistance.

1. Introduction

Adhesion coefficient of the locomotive wheels with the rails ψ_s should be determined based on the empirical results of the locomotive performance, while its value taken for traction calculation should ensure the most effective use of the locomotive power and steady traffic of freight trains. This problem has considerably grown in importance, since the number of heavy (4 000–5 000 tons) freight trains has largely increased on Lithuanian railways in recent years (2007–2008 m. prekinių traukinių... 2007).

The use of methods of mathematical statistics and theory of probability provides wider possibilities for determining the most acceptable (suitable) value of the adhesion coefficient ψ_{adh}. Their application allows us to determine not only the average values of the coefficient ψ_s, but to estimate the whole range of experimental results. However, usually, only the values of the adhesion coefficient exceeding its average values were determined by Bureika and Mikaliūnas (2002). Under the conditions of the intensive use of locomotives, it is of paramount importance to analyze and precisely evaluate factors influencing the reduction of the coefficient ψ_{adh}. When, in calculating the locomotive speed, the adhesion coefficient is represented by a single curve $\psi_{adh}(v)$ and a single value ψ_{adh}, though the external factors of its maintenance are changing in a wide range, this can hardly ensure the most rational choice of the tractive force of a locomotive depending on the adhesion value F_{tr}^{calc} and the locomotive mass for a particular route. The use of the adhesion coefficient values based on reliable research results could help maintain uninterrupted (continuous) traffic of heavy freight trains and increase not only the carrying capacity, but also the volume of the transported goods (Handbook… 2006).

To determine the most suitable adhesion coefficient ψ_{adh} value for particular operating conditions, the factors influencing this value should be analysed. On Lithuanian railways, the main problem for heavy freight trains is running uphill incline of 8.1 % and 3 km-long track gradient on the railway line between the stations of Telšiai and Lieplaukė in Lithuania. The paper considers a mathematical model of regular traffic of heavy freight trains pulled by upgraded freight locomotives of the series 2M62M on this line.

2. Random factors influencing steady train motion uphill

The locomotive, rolling stock and the road make a complicated mechanical system influenced by a great number of random external and internal factors, e.g. unevenness of the rails or non-uniformly worn-out wheel-set rolling surfaces as well as different rolling radii of one of

ISSN 1648-4142 print / ISSN 1648-3480 online
www.transport.vgtu.lt
DOI: 10.3846/1648-4142.2008.23.135–137

the wheel-set tyres, variation of the road curve radius, dirty or wet rails, interaction of wagons, etc. Two main concepts concerning train motion up the vertical grade may be defined as continuous and non-continuous (unsteady) motion. When the motion is continuous, the speed of the train v_{calc} can be calculated irrespective of the influence of random factors. When the motion of the train is non-continuous (unsteady), its speed cannot be maintained as specified and decreases, though the locomotive operator takes some actions to control it.

The condition of steady train motion uphill is expressed as follows:

$$F_{tr}^{adh} \geq W_j \quad \text{and} \quad v_{steady} \geq v_{calc}, \tag{1}$$

where: F_{tr}^{adh} is tractive force according to adhesion, kN; W_j is running resistance, kN; v_{steady} is train speed in steady motion, km/h; v_{calc} is the calculated train velocity, km/h.

If the conditions of wheel adhesion with the rails ensure the application of the tractive force equal to or exceeding the total resistance, train motion is continuous or steady.

The curves showing the dependence of the specified calculated tractive force F_{tr}^{calc}, actual tractive force based on the adhesion F_{tr}^{adh} and resistance to motion W_j on the train speed v are presented in Fig. 1.

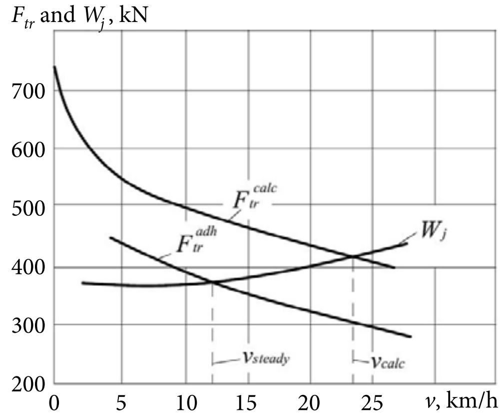

Fig. 1. The dependence of calculated tractive force F_{tr}^{calc}, actual tractive force based on adhesion F_{tr}^{adh} and running resistance W_j on the locomotive velocity v: v_{steady} is locomotive travelling steady velocity; v_{calc} is calculated velocity

The intersection point A of the curves of the tractive force F_{tr}^{calc} and resistance W_j (Fig. 1) indicates the calculated velocity v_{calc} of the train. If the actually applied tractive force based on the adhesion F_{tr}^{adh} is lower than the calculated tractive force F_{tr}^{calc}, then, the values of running resistance W_j are decreasing according to the respective law, when the train speed is decreasing and the adhesion force F_{adh} is increasing. When a certain value of the decreasing speed v_{steady}, is reached, the equilibrium between the locomotive tractive force and running resistance is established, and the motion becomes steady. The transient process is over, and the train goes further uphill at a lower speed.

Thus, given the train mass m_{train}, and the defined dependence $W_j(v)$, in each case, the actual adhesion force F_{tr}^{adh} corresponds to the altered steady speed value. In some critical situations, the unsteady mode of locomotive motion is characterized by speed reduction to 0, i. e. the train stops "stretched", implying that there are no gaps between the automatic clutches of the train wagons. These situations are possible but rarely found in practice.

3. Determining the conditions of steady locomotive motion

A mode of locomotive motion largely depends on the relationship between the total train running resistance W_j and tractive force F_{tr}^{adh}. Therefore, the problem of determining steady motion conditions is the analysis of the system of two random quantities – F_{tr}^{adh} and W_j. The distribution of their values is determined by a large number of factors which are not closely interrelated, each of them making a small contribution to the joint effect. This allows us to assume that the quantities F_{tr}^{adh} and W_j are distributed according to the normal law. Similar assumptions are made by Grebenyuk (Гребенюк 2003) when investigating locomotive traction.

The quantities F_{tr}^{adh} and W_j are the speed functions:

$$F_{tr}^{adh}(v) = 1000 \cdot P \cdot g \cdot \Psi_{adh}(v), \tag{2}$$

$$W_j(v) = Q \cdot g \cdot \left(i + w_0''(v)\right) + P \cdot g \cdot \left(i + w_0'(v)\right), \tag{3}$$

where: P is total mass of locomotive, t; Q is total mass of wagons, t; i is uphill gradient, %; g is gravitational acceleration, m/s^2; w_0' is the locomotive's running main relative resistance, N/t; w_0'' is the wagons' running main relative resistance, N/t.

Adhesion coefficients of locomotives and the main relative running resistance are found by the empirical formulas, see researches by Wickens (2003) and Grebenyuk (Гребенюк 2003):

$$\Psi_{adh} = a + \frac{b}{c + d \cdot v} - e \cdot v; \tag{4}$$

$$w_0' = l + p \cdot v + f \cdot v^2; \tag{5}$$

$$w_0'' = h + \frac{r + m \cdot v + n \cdot v^2}{q_a}, \tag{6}$$

where: $a, b, c, d, e, f, h, l, m, n, p, r$ are constant empirical coefficients of particular trains; q_a is wagon axle load, tf.

Based on the actual values of the adhesion coefficient ψ_{adh} and the specified calculated speed v_{calc}, the mass of the train moving at steady speed along the uphill's determinant gradient i_{det} is calculated for a locomotive of the considered series according to the condition $F_{tr}^{adh} = W_j$. In this case, average values of the main relative resistance w_0'' and w_0' are used:

$$(P + Q) = \frac{1000 \cdot (P \cdot g) \cdot \Psi_{adh} - (P \cdot g) \cdot (w_0' + i_{det})}{w_0'' + i_{det}}. \tag{7}$$

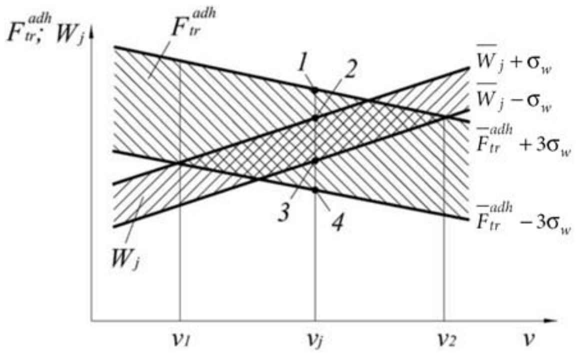

Fig. 2. Tractive forces based on adhesion F_{tr}^{adh} and the variation range of actual total resistance W_j values depending on locomotive motion speed v: in the range between point 1 and point 2 – steady (continuous) motion; between point 3 and point 4 – unsteady (non-continuous) motion; between point 2 and point 3 – indeterminate train's motion

Here, the specified (calculated) values of F_{tr}^{adh} and W_j are taken. In fact, these are random quantities, therefore, two ranges of values, rather than two speed dependence values, are obtained (see Fig. 2).

The limits of the variation range of force values are defined, taking into account mathematical expectation dependence on speed and standard deviations σ_F, σ_W. According to the predetermined probability value, the expected values of F_{tr}^{adh} and W_j are within these ranges.

In each case, the locomotive adhesion tractive force F_{tr}^{adh} in the range of values from $F_{tr}^{adh} - \sigma_F$ to $F_{tr}^{adh} + \sigma_F$ can be applied. At the same time, the actual tractive force F_{tr}^{act}, can be equal to the total resistance W_j or be smaller or larger than its value. When the minimum tractive force (the lower variation range limit of F_{tr}^{adh}) is used at the maximum total resistance (the upper limit of the values' variation range), the steady speed will be v_2. Thus, for any ratio of tractive force to total resistance, the travelling speed of a locomotive will be between v_1 and v_2. Therefore, steady train motion can be only achieved in the above-mentioned speed range between v_1 and v_2.

4. Developing a mathematical model of steady running uphill of the train

The calculated speed v_{calc} provided will be used, when tractive force based on the value of adhesion F_{tr}^{adh} will be in the range between points 1 and 2 (Fig. 2). If the value F_{tr}^{adh} is found between the points 3 and 4, the running of the train will be unsteady. In the range from point 2 to point 3, running stability depends on the ratio of force F_{tr}^{adh} to resistance W_j. A criterion describing the stability of running may be the difference between the above forces Δ_{calc}, computed for the calculated velocity:

$$\Delta_{calc} = F_{tr}^{adh}\left(v_{calc}\right) - W_j\left(v_{calc}\right). \qquad (8)$$

If $\Delta_{calc} \geq 0$, the running of the train is steady. Otherwise, the steady travelling speed of the train will be lower than the calculated speed v_{calc}.

In general, smoothly adjusting the tractive force and speed of a locomotive, we can achieve a tractive force corresponding to actual dependence of $F_{tr}^{adh}(v)$ Then, the steady speed v_{steady} can be expressed as:

$$v_{steady} = f\left(F_{tr}^{adh}(v) - W_j\left(v\right)\right) = f\left(v\right),$$

$$\Delta = F_{tr}^{adh} - W_j\left(v\right). \qquad (9)$$

The above steady (uniform) speed v_{steady} is the function of the difference Δ. By making some substitutions, we will obtain a 3rd order equation $a_0 \cdot v_{steady}^3 + a_1 \cdot v_{steadyl}^2 + a_2 \cdot v_{steady} = 0$, which should be solved.

When adhesion coefficient and random deviations from the total running resistance are not taken into consideration, the calculated speed and weight of the train as well as time intervals between the running trains are not optimal. As a result, the train schedule can hardly be reliable in this case.

5. Conclusions

1. Solving the problem of steady train running uphill, its tractive force based on adhesion and the total resistance should be analysed as random quantities, using the methods of mathematical statistics and theory of probability.
2. The research results show that the actual values of the adhesion coefficient Ψ_{adh} and the total motion resistance W_j are distributed according to the normal distribution law, while the actual values differ from the specified calculated values by about 3–4 %.
3. When specifying the actual calculated velocity v_{calc}^{act} for Lithuanian freight trains running Lieplaukė uphill gradient of 8.1 %, it should be decreased by about 8–10 %. This could ensure steady running of freight trains uphill as well as steady traffic between these stations.

References

Bureika, G.; Mikaliūnas, Š. 2002. Peculiarities of traction forces in wheel/rail contact area, *Transport* 17(1): 8–14.

2007–2008 m. prekinių traukinių eismo tvarkaraštis. Galioja nuo 2007 m. gegužės 27 d. [The schedule of freight trains for 2007–2008. Valid from 27 May, 2007]. Vilnius: UAB „Lodvila", 2007. 208 p. (in Lithuanian).

Handbook of Railway Vehicle Dynamics. Edited by Simon Iwnicki. Taylor & Francis Group, 2006. 527 p.

Wickens, A. H. 2003. *Fundamentals of rail vehicle dynamics: guidance and stability*. Swets & Zeitlinger, Lisse, The Netherlands. ISBN 902651946X.

Гребенюк, П. Т. 2003. *Продольная динамика поезда* [Grebenyuk, P. T. Longitudinal train dynamics]. Москва: Интекст. 96 с. (in Russian).