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FORECASTING THE REAL AVERAGE RETIREMENT BENEFIT IN THE UNITED STATES USING OWA OPERATORS

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Abstract. The issue of pensions has become increasingly topical. This paper presents the ordered weighted averaging real average pension (OWARAP) operator. The OWARAP operator is based on the ordered weighted averaging (OWA) operator and calculates the future average retirement benefit taking into account price changes. Moreover, this work extends the OWARAP operator by using order-inducing variables, generalized means, and probabilities. This paper ends by analyzing the applicability of the OWARAP operator and its extensions in forecasting the real average Social Security benefits for retired workers in each state of the United States (U.S.). The results demonstrate the usefulness of the proposed approach in retirement decision making.

Keywords: aggregation operator, forecasting, inflation, OWA operator, retirement benefit, Social Security.

JEL Classification: C53, D81, H55.

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1. Introduction

The continuous growth in life expectancy, partly driven by an improvement in healthcare systems, and the low fertility rates imply an increase in the old-age dependency ratio (that is, the number of elderly people compared to those at working age). Consequently, retirement systems become more unsustainable (Organization for Economic Cooperation and Development [OECD], 2019; Peris-Ortiz et al., 2020), which also has a negative impact on the retirement income adequacy. For example, in 1990, life expectancy at age 65 was 18.9 years for woman and 15.1 years for men in the United States (U.S.) (OECD, 2023b). Thirty years later, in 2020, life expectancy at age 65 increased considerably to 19.8 years for women and 17.0 years for men. Also, by looking at the fertility rates for the same country, one can see that the fertility rate declined from 2.08 children per women in 1990 to 1.64 in 2020 (OECD, 2023a). However, these demographic changes are not the only ones that adversely affect the financial health of retirement systems and, consequently, the adequacy levels. There are other factors related to economic growth, the labor market, and the design of the retirement system (OECD, 2019; Peris-Ortiz et al., 2020), among others.

In this complex context, it is very important that citizens are provided with sufficient and recurrent retirement information (Basiglio & Oggero, 2020). For instance, individuals need to be aware of their future retirement income as accurately as possible and in real prices

(Bongini & Cucinelli, 2019; O'Neill et al., 2017) so that they can properly plan their retirement and avoid a reduction in purchasing power. Likewise, governments need access to precise and helpful information on the future trend of public retirement benefits and other related indicators to conduct effective policy decision making and thereby reduce the risk of poverty among older people. However, there is a high level of uncertainty associated with this information that needs to be managed. Numerous tools have been developed to handle uncertainty present in different situations (Figuerola-Wischke et al., 2022; Su et al., 2013; Zeng, Gu, et al., 2023; Zeng, Hu, et al., 2023).

With the purpose of helping individuals to have an adequate amount of savings for their retirement, and also governments to make good decisions, this paper presents the ordered weighted averaging real average pension (OWARAP) operator. The OWARAP operator can be seen as an enhanced retirement index. It is built under the ordered weighted averaging (OWA) operator from Yager (1988) while considering the effect of inflation. The OWA operator is an increasingly popular aggregation operator used for fusing numerical information based on the attitudinal character of the decision maker by generating weights (Emrouznejad & Marra, 2014; He et al., 2017; Yu et al., 2023). Thus, this approach allows to overestimate or underestimate the real average retirement benefit according to the opinion of the decision maker, that is, considering the bipolar preferences, which is very useful for dealing with demographic, economic, and pension policy uncertainties.

Over the last years, a great number of researchers have proposed different extensions of the OWA operator. Some of the most notable are the induced OWA (IOWA) operator (Yager & Filev, 1999), the generalized OWA (GOWA) operator (Yager, 2004), and the probabilistic OWA (POWA) operator (Merigó, 2009, 2012). Specifically, the IOWA operator uses order-inducing variables; the GOWA operator generalized means (Dyckhoff & Pedrycz, 1984); and the POWA operator probabilistic information. This study considers these extensions in the OWARAP operator, thus obtaining the induced OWARAP (IOWARAP) operator, the generalized OWARAP (GOWARAP) operator, and the probabilistic OWARAP (POWARAP) operator. This allows the decision maker to contemplate a diverse range of aggregation operators and adopt the one that best suits with his/her needs and preferences.

In the literature, we can find various authors that apply the OWA operator and extensions of this operator in economic indicators, including exchange rates (Flores-Sosa et al., 2020; León-Castro et al., 2016, 2018), inflation rates (Espinoza-Audelo et al., 2020; León-Castro et al., 2020), and prosperity (Amin & Siddiq, 2019). However, they have not yet been applied to the average retirement benefit. Therefore, the study's novelty consists of using the characteristics of the OWA operator to forecast the average retirement benefit adjusted for inflation.

This paper is organized as follows. Section 2 briefly reviews some basic but necessary concepts. Section 3 explains the mathematical framework used in this work. Section 4 develops an exhaustive numerical example of the proposed approach, which consists in forecasting the real average Social Security retirement benefit of each state of the U.S. The last section summarizes the main conclusions of the paper and makes some general recommendations for future research.

2. Preliminaries

The following section briefly reviews the OWA operator, the IOWA operator, the GOWA operator, and the POWA operator.

2.1. The OWA operator

The OWA operator was presented by Yager (1988) and it provides a parameterized class of mean type aggregation operators that lie between the minimum and the maximum. During the last three decades, this operator has been successfully applied in a large variety of fields (Kacprzyk et al., 2019). The OWA operator can be defined as follows.

Definition 1. An OWA operator of dimension n is a mapping $OWA{:}R^n \rightarrow R$ that has associated a weighting vector $W = (w_1, ..., w_n)$, with $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$, such that:

$$OWA(a_1, \ldots, a_n) = \sum_{j=1}^{n} w_j b_j, \tag{1}$$

where b_j is the jth largest element of the arguments $a_1, ..., a_n$, namely $(b_1, ..., b_n)$ is $(a_1, ..., a_n)$ reordered in a descending way.

The parameterization is carried out by choosing different formations of the weighting vector W. For example, if $w_j = 1/n$, for all j, the Laplace criterion (also known as arithmetic mean) is formed. Furthermore, another aspect worth highlighting is that when the reordering process is conducted in an ascending manner, then we get the ascending OWA (AOWA) operator (Yager, 1992).

2.2. The IOWA operator

A remarkable extension of the OWA operator is the IOWA operator (Yager & Filev, 1999). The main difference between this operator and the classical OWA operator is that the reordering step is carried out with order-inducing variables. This is why a major advantage of the IOWA operator is that it can consider the complex attitudes of the decision maker. This operator can be defined as follows.

Definition 2. An IOWA operator of dimension n is a mapping $IOWA{:}R^n \times R^n \rightarrow R$ that has associated a weighting vector $W = (w_1, ..., w_n)$, with $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$, such that:

$$IOWA(\langle u_1, a_1 \rangle, \ldots, \langle u_n, a_n \rangle) = \sum_{j=1}^{n} w_j b_j, \tag{2}$$

where b_j is the a_i value of the IOWA pair $\langle u_i, a_i \rangle$ having the jth largest u_i value, u_i is referred as the order-inducing variable, and a_i is the argument variable.

2.3. The GOWA operator

The GOWA operator was introduced by Yager (2004), combining the OWA operator with generalized means (Dyckhoff & Pedrycz, 1984). Specifically, this operator incorporates a parameter that allows control of the power to which the argument values are raised in the aggregation. The GOWA operator can be defined as follows.

Definition 3. A GOWA operator of dimension n is a mapping $GOWA{:}R^n \to R$ that has associated a weighting vector $W = (w_1, \ldots, w_n)$, with $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$, such that:

$$GOWA\left(a_1,\ldots,a_n\right) = \left(\sum_{j=1}^{n} w_j b_j^{\lambda}\right)^{1/\lambda}, \tag{3}$$

where λ is a controlling parameter that may take any value in the interval (–∞, ∞) and b_j is the jth largest of the argument variable a_i.

Recall that: if λ = –1, the ordered weighted harmonic averaging (OWHA) operator (Yager, 2004) is found; if λ = 0, the ordered weighted geometric (OWG) operator (Chiclana et al., 2000, 2002); if λ = 1, the ordinary OWA operator; and if λ = 2, the ordered weighted quadratic averaging (OWQA) operator (Yager, 2004).

2.4. The POWA operator

The POWA operator was presented by Merigó (2009, 2012). It can be described as an aggregation operator that integrates both the OWA operator and the probability in the same formulation and based on the level of importance of these two concepts in the aggregation procedure. As a result, it provides an integrated system for decision making problems under risk and uncertainty. The POWA operator can be defined as follows.

Definition 4. A POWA operator of dimension n is a mapping $POWA{:}R^n \to R$ that has associated a weighting vector $W = (w_1, \ldots, w_n)$ with $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$, such that:

$$POWA\left(a_1,\ldots,a_n\right) = \sum_{j=1}^{n} \hat{v}_j b_j = \beta \sum_{j=1}^{n} w_j b_j + \left(1-\beta\right) \sum_{i=1}^{n} v_i a_i, \tag{4}$$

where b_j is the jth largest of the argument a_i, each argument a_i has associated probability v_i with $\sum_{i=1}^{n} v_i = 1$ and $v_i \in [0,1]$, $\hat{v}_j = \beta w_j + \left(1-\beta\right) v_j$ with $\beta \in [0,1]$, and v_j is the probability v_i ordered according to b_j, that is, based on the jth largest of the argument a_i.

3. OWA operators in the real average pension benefit

In the following section, the OWARAP operator and some of its extensions and generalizations will be defined and analyzed.

3.1. The OWARAP operator

The OWARAP operator is an aggregation operator based on Yager's OWA operator. In particular, it aggregates the information of a set of nominal average retirement benefits and another one with inflations while considering the attitude, judgment, or knowledge of the decision maker. This feature makes the OWARAP operator an attractive method for forecasting the real average retirement benefit under uncertainty, as the decision maker is capable of overestimating or underestimating the projections. Also, by making inflation adjustments, individuals are able to control if their retirement benefits will or will not grow in the future at the same pace as inflation does. This operator is defined as follows.

Definition 5.1. An OWARAP operator of dimension n is a mapping $OWARAP{:}R^n \times R^n \to R$ that has associated a weighting vector $W = (w_1, \ldots, w_n)$ with $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$, in which:

$$OWARAP\left(\langle CPI_1, p_1\rangle, \ldots, \langle CPI_n, p_n\rangle\right) = \sum_{j=1}^{n} w_j P_j, \tag{5.1}$$

where P_j is the jth largest of the $\left(\frac{100}{CPI_i}\right)p_i$, p_i is the ith argument of a set of nominal average retirement benefits, and CPI_i is the ith argument of a set of consumer price indices.

Note that Definition 5.1 contemplates different CPI input values in the aggregation. However, if the decision maker wants to consider a single CPI input value, then the mathematical expression of the OWARAP operator can be rewritten as follows.

Definition 5.2. An OWARAP operator of dimension n is a mapping $OWARAP{:}R^n \to R$ that has associated a weighting vector $W = (w_1, \ldots, w_n)$ with $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$, in which:

$$OWARAP\left(p_1, \ldots, p_n\right) = \left(\frac{100}{CPI}\right)\sum_{j=1}^{n} w_j P_j, \tag{5.2}$$

where P_j is the jth largest argument of a set of nominal average retirement benefits $p_1, \ldots, p_n$ and CPI is the consumer price index.

Henceforth, the study assumes that several scenarios for the CPI may be possible.

Example 1. Consider the following set of nominal retirement benefits $\left(p_1 = 1{,}300, p_2 = 1{,}250, p_3 = 1{,}350, p_4 = 1{,}400\right)$ and weighting vector $\left(w_1 = 0.4, w_2 = 0.3, w_3 = 0.2, w_4 = 0.1\right)$. If the collection of consumer prices indices is $\left(CPI_1 = 190, CPI_2 = 180, CPI_3 = 200, CPI_4 = 210\right)$, then, the aggregation process through the OWARAP operator, that is, Eq. (5.1), is solved as follows:

$$0.4 \times \frac{100}{180} \times 1{,}250 + 0.3 \times \frac{100}{190} \times 1{,}300 + 0.2 \times \frac{100}{200} \times 1{,}350 + 0.1 \times \frac{100}{210} \times 1{,}400 = 684.7.$$

The OWARAP operator is an averaging operator that fulfils the properties of monotonicity, commutativity (also referred to as symmetry or anonymity), boundedness, and idempotency (also called agreement or unanimity). These properties are explained below with their corresponding theorems. Take into account that most of the proofs are omitted as they are considered trivial and repetitive.

Theorem 1. Monotonicity. It states that when an argument increases, the final aggregation remains equal or increases, but in no case decreases. If $\left(\frac{100}{CPI_i}\right)p_i \geq \left(\frac{100}{\widehat{CPI}_i}\right)\hat{p}_i$, for all i, then, $OWARAP\left(\langle CPI_1, p_1\rangle, \ldots, \langle CPI_n, p_n\rangle\right) \geq OWARAP\left(\langle \widehat{CPI}_1, \hat{p}_1\rangle, \ldots, \langle \widehat{CPI}_n, \hat{p}_n\rangle\right)$.

Theorem 2. Commutativity. Meaning that the initial ordering of the arguments is completely irrelevant. Thus, $OWARAP\left(\left\langle CPI_1,p_1\right\rangle,\ldots,\left\langle CPI_n,p_n\right\rangle\right)=OWARAP\left(\left\langle \widehat{CPI}_1,\hat{p}_1\right\rangle,\ldots,\left\langle \widehat{CPI}_n,\hat{p}_n\right\rangle\right)$, where $\left(\left\langle \widehat{CPI}_1,\hat{p}_1\right\rangle,\ldots,\left\langle \widehat{CPI}_n,\hat{p}_n\right\rangle\right)$ is any permutation of $\left(\left\langle CPI_1,p_1\right\rangle,\ldots,\left\langle CPI_n,p_n\right\rangle\right)$.

Theorem 3. Boundedness. In the sense that the aggregation is delimited. Accordingly, $Min\left\{\left(\frac{100}{CPI_i}\right)p_i\right\}\le OWARAP\left(\left\langle CPI_1,p_1\right\rangle,\ldots,\left\langle CPI_n,p_n\right\rangle\right)\le Max\left\{\left(\frac{100}{CPI_i}\right)p_i\right\}$.

Theorem 4. Idempotency. It signifies that if all the input arguments are the same, then the aggregated output should match with them. If $\left(\frac{100}{CPI_i}\right)p_i=\left(\frac{100}{CPI}\right)p$, for all i, then, $OWARAP\left(\left\langle CPI_1,p_1\right\rangle,\ldots,\left\langle CPI_n,p_n\right\rangle\right)=\left(\frac{100}{CPI}\right)p$.

Proof. Since $p_i = p$ and $CPI_i = CPI$, for all i, we have

$$OWARAP\left(\left\langle CPI_1,p_1\right\rangle,\ldots,\left\langle CPI_n,p_n\right\rangle\right)=\sum_{j=1}^{n}w_jP_j=\sum_{j=1}^{n}w_j\left(\frac{100}{CPI}\right)p=\left(\frac{100}{CPI}\right)p\sum_{j=1}^{n}w_j.$$

Since $\sum_{j=1}^{n}w_j=1$, we get

$$OWARAP\left(\left\langle CPI_1,p_1\right\rangle,\ldots,\left\langle CPI_n,p_n\right\rangle\right)=\left(\frac{100}{CPI}\right)p.$$

This operation can be carried out multiple times without changing the result, therefore, it can be stated that the OWARAP operator is idempotent.

Furthermore, to determine the values of the weighting vector W of the OWARAP operator, it is possible to use the well-known characterizing measures presented by Yager and Alajlan. These measures are the degree of orness (Yager, 1988), the entropy of dispersion (Yager, 1988), the balance operator (Yager, 1996), the divergence (Yager, 2002), and the focus (Yager & Alajlan, 2014).

The degree of orness measure, also referred to as the attitudinal character, can be defined as follows:

$$\alpha\left(W\right)=\sum_{j=1}^{n}w_j\left(\frac{n-j}{n-1}\right). \tag{6}$$

The entropy of dispersion measure can be defined as follows:

$$H\left(W\right)=-\sum_{j=1}^{n}w_j\ln\left(w_j\right). \tag{7}$$

The balance operator measure can be defined as follows:

$$Bal\left(W\right)=\sum_{j=1}^{n}w_j\left(\frac{n+1-2j}{n-1}\right). \tag{8}$$

The divergence measure can be defined as follows:

$$Div\left(W\right)=\sum_{j=1}^{n}w_j\left(\frac{n-j}{n-1}-\alpha\left(W\right)\right)^2. \tag{9}$$

And the focus measure can be defined as follows:

$$Focus(W) = 1 - \frac{2}{n}\sum_{j=1}^{n} w_j \left|m - j\right|, \tag{10}$$

where $m = n\left(1 - \alpha(W)\right) + \alpha(W)$.

3.2. Families of the OWARAP operator

Different families of the OWARAP operator can be obtained by choosing different manifestations of the weighting vector W. In the following, some of these families are presented.

- When $w_1 = 1$ and $w_j = 0$, for all $j \neq 1$, the maximum OWARAP operator is found, which corresponds to the optimistic decision criterion.
- When $w_n = 1$ and $w_j = 0$, for all $j \neq n$, the minimum OWARAP operator is found, which corresponds to the pessimistic decision criterion.
- If n is an odd number, then, when $w_{(n+1)/2} = 1$ and $w_j = 0$, for all $j \neq (n+1)/2$, the median OWARAP operator is formed. Otherwise, in the case that n is even, the median OWARAP operator is obtained when $w_{n/2} = w_{(n/2)+1} = 0.5$ and $w_j = 0$, for all $j \neq n/2$, $(n/2) + 1$.
- When $w_j = 1/n$, for all j, the normalized OWARAP operator is found, which corresponds to the Laplace decision criterion, that is, the arithmetic mean.
- When $w_1 = \alpha$, $w_n = 1 - \alpha$, and $w_j = 0$, for all $j \neq 1, n$, the Hurwicz OWARAP operator is found.
- When $w_1 = w_n = 0$ and $w_j = 1/(n - 2)$, for all $j \neq 1, n$, the Olympic OWARAP operator is found.
- When $w_k = 1$ and $w_j = 0$, for all $j \neq k$, the step OWARAP operator is found.

3.3. Extensions and generalizations of the OWARAP operator

An interesting extension of the OWARAP operator is the IOWARAP operator, which uses order-inducing variables in the process of reordering the set of values $\left(\left\langle CPI_1, p_1\right\rangle, \ldots, \left\langle CPI_n, p_n\right\rangle\right)$. Thus, the reordering step does not depend on the values of the arguments p_i and CPI_i. This is why the main advantage of this extension is the possibility to consider more complex attitudes of the decision maker. The IOWARAP operator is defined as follows.

Definition 6. An IOWARAP operator of dimension n is a mapping $IOWARAP: R^n \times R^n \times R^n \to R$ that has associated a weighting vector $W = (w_1, \ldots, w_n)$ with $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$, in which:

$$IOWARAP\left(\left\langle u_1, CPI_1, p_1\right\rangle, \ldots, \left\langle u_n, CPI_n, p_n\right\rangle\right) = \sum_{j=1}^{n} w_j P_j, \tag{11}$$

where p_j is the $\left(\frac{100}{CPI_i}\right)p_i$ value of the IOWARAP triplet $\left\langle u_1, CPI_1, p_1\right\rangle$ having the jth largest u_i value. u_i is referred as the order-inducing variable, p_i as the nominal average retirement benefit variable, and CPI_i as the consumer price index variable.

Moreover, by incorporating generalized means in the OWARAP operator, the GOWARAP operator is obtained. Specifically, it adds a parameter that controls the power to which the argument values are raised. Thus, this operator comprises an extensive range of aggregation operators, including the OWARAP operator and its particular cases, among others. The GOWARAP operator is defined as follows.

Definition 7. A GOWARAP operator of dimension n is a mapping $GOWARAP{:}R^n \times R^n \rightarrow R$ that has associated a weighting vector $W = (w_1, \ldots, w_n)$ with $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$, in which:

$$GOWARAP\left(\langle CPI_1, p_1\rangle, \ldots, \langle CPI_n, p\rangle_n\right) = \left(\sum_{j=1}^{n} w_j P_j^{\lambda}\right)^{1/\lambda}, \quad (12)$$

where P_j is the jth largest of the $\left(\frac{100}{CPI_i}\right)p_i$ and λ is a controlling parameter that may take any value in the interval $(-\infty, \infty)$. p_i is the nominal average retirement benefit variable and CPI_i the consumer price index variable.

By giving different values to the controlling parameter λ, it is possible to find particular cases of the GOWARAP operator, among which the following are noteworthy:

- When $\lambda = -1$, the harmonic OWARAP (OWHARAP) operator is formed.
- When $\lambda = 0$, the geometric OWARAP (OWGRAP) operator is formed.
- When $\lambda = 1$, the OWARAP operator is formed.
- When $\lambda = 2$, the quadratic OWARAP (OWQARAP) operator is formed.

Likewise, by analyzing the weighting vector W and the controlling parameter λ jointly, it can be summarized that:

- When $\lambda = -\infty$ and $w_n \neq 0$, the smallest $(100/CPI_i)p_i$ value of the collection is obtained, which is P_n.
- When $\lambda = -\infty$ and $w_1 = 1$, that is, $w_j = 0$, for all $j \neq 1$, the largest $(100/CPI_i)p_i$ value of the collection is achieved, which is P_1.
- When $\lambda = \infty$ and $w_1 \neq 0$, the largest $(100/CPI_i)p_i$ value of the collection is obtained, which is P_1.
- When $\lambda = \infty$ and $w_n = 1$, that is, $w_j = 0$, for all $j \neq n$, the smallest $(100/CPI_i)p_i$ value of the collection is achieved, which is P_n.

Another appealing aggregation operator is the POWARAP operator, which unifies the probability and the OWARAP operator into a single formulation. Hence, it adds more information to the final outcome. With this operator, it is possible to overestimate or underestimate the probabilities based on the attitudinal character of the decision maker. The POWARAP operator is defined as follows.

Definition 8. A POWARAP operator of dimension n is a mapping $POWARAP{:}R^n \times R^n \rightarrow R$ that has associated a weighting vector $W = (w_1, \ldots, w_n)$ with $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$, in which:

$$POWARAP\left(\langle CPI_1, p_1\rangle, \ldots, \langle CPI_n, p_n\rangle\right) = \sum_{j=1}^{n} \hat{v}_j P_j = \beta \sum_{j=1}^{n} w_j P_j + (1-\beta) \sum_{i=1}^{n} v_i \left(\frac{100}{CPI_i}\right) p_i, \quad (13)$$

where P_j is the jth largest of the $\left(\frac{100}{CPI_i}\right)p_i$, p_i is referred as the nominal average retirement benefit variable, and CPI_i is the consumer price index variable. Each $\left(\frac{100}{CPI_i}\right)p_i$ has associated probability v_i with $\sum_{i=1}^{n} v_i = 1$ and $v_i \in [0,1]$, $\hat{v}_j = \beta w_j + (1-\beta) v_j$ with $\beta \in [0,1]$, and v_j is the probability v_i ordered according to P_j, that is, based on the jth largest of the $\left(\frac{100}{CPI_i}\right)p_i$.

Observe that with the parameter β, the decision maker can represent the degree of importance that the OWARAP operator and the probability have in the aggregation process. For example, when the parameter β is equal to 1, the OWARAP operator is obtained, which means that the decision maker does not consider probabilistic information. Conversely, when β is equal to 0, the expected value is gotten, meaning that full importance is given to the probability.

4. Forecasting the U.S. real average Social Security retirement benefit

Retirement income in the U.S. is based upon three pillars (Kintzel, 2017): Social Security, employer-sponsored plans, and personal savings. The following section focuses solely on the first one. However, it is worth to briefly review each of them in order to get a general idea of the U.S. retirement system.

The Old-Age, Survivors, and Disability Insurance (OASDI), or simply known as Social Security, is a program run by the federal government of the U.S., more specifically the Social Security Administration [SSA]. It is financed through payroll taxes on employers, employees, and self-employed. Social Security provides different types of benefits, although the largest part is dedicated to the payment of retirement benefits to retired workers. Moreover, OASDI benefits are annually adjusted for inflation based on the CPI for urban wage earners and clerical workers (CPI-W) not seasonally adjusted (NSA) (SSA, 2021). This is known as cost-of-living adjustment (COLA).

We can distinguish between two types of employer-sponsored plans. On the one hand, there are defined benefit (DB) plans. On the other hand, there are defined contribution (DC) plans, where the most popular type is the 401(k). Furthermore, over the last decades, there has been a significant shift from DB to DC plans (Altman & Kingson, 2021; Rauh et al., 2020). However, DC plans are less secure than DB ones because the investment risk is placed on the individuals.

Additionally, workers can also individually save for their retirement. One common way of doing this is through an individual retirement account (IRA), which can be simply described as an investment account with tax advantages.

In the following, an illustrative example of the explained approach is developed for forecasting the December 2025 real average Social Security retirement benefit paid to a retired worker in each state of the U.S. Moreover, a multi-expert analysis will be adopted to provide a more representative view of the problem. This numerical example is divided into five explanatory steps, the assessment of the final results, and a comparative exercise.

Step 1. First, historical data regarding the number of Social Security beneficiaries and the amount of Social Security benefits paid to retired workers in the U.S. by state is collected. Afterward, the amount of benefits is divided by the number of beneficiaries in order to obtain the average benefit. These data were extracted from the SSA database; however, with the limitation that only annual data as value at end of period, that is, December, was available. Similarly, historical COLA data was retrieved from the same source.

Data about the CPI for all urban consumers (CPI-U) NSA with base period 1982–1984 was also gathered, but in this case, from the U.S. Bureau of Labor Statistics (BLS). Note that when computing the average benefits in real terms, the CPI-U is used instead of the CPI-W. The CPI for the elderly (CPI-E) is not used either. The nominal and real average benefits for December 2021 (latest available data) can be seen in Table 1.

Step 2. Once all the data has been collected, three experts (e_1, e_2, e_3) are asked to provide their individual estimations of the COLA and CPI-U NSA development for the years 2023 to 2025. Table 2 shows this information.

Step 3. Then, the nominal average Social Security benefits for retired workers can be forecasted through the application of a simple linear regression with the COLA as the independent variable. Note that the dependent variable is expressed in terms of growth. For each state the coefficient of determination (R^2) is greater than 0.9, meaning that the model has a highly good fit. Three different forecast scenarios (S_1, S_2, S_3) are obtained based on the inputs provided by the experts (see Table 3).

Step 4. Next, the weighting vector W, inducing vector U, and probabilistic vector V are defined as follows: W = (0.7, 0.2, 0.1), U = (7, 9, 8), and V = (0.2, 0.5, 0.3). Note that subjective probabilities are considered. Likewise, the parameter β is determined as follows: β = 0.5.

Step 5. Lastly, the forecast scenarios calculated in Step 3 are combined into a single result. To make this, the OWARAP operator, the AOWARAP operator, the IOWARAP operator, the OWHARAP operator, the OWQARAP operator, and the POWARAP operator are used. Table 4 presents the final results.

Table 1. December 2021 nominal and real average Social Security benefits for retired workers (benefits in dollars)

	State	Total beneficiaries	Total benefits (thousands)	Avg. benefits (nominal)	Avg. benefits (real)
NR	Connecticut	532,298	975,916	1,833	658
	Maine	258,610	405,736	1,569	563
	Massachusetts	939,694	1,625,073	1,729	620
	New Hampshire	236,601	426,404	1,802	646
	New Jersey	1,244,222	2,283,490	1,835	658
	New York	2,684,406	4,583,696	1,708	612
	Pennsylvania	2,088,154	3,591,417	1,720	617
	Rhode Island	167,529	286,073	1,708	612
	Vermont	116,636	197,566	1,694	608

End of Table 1

	State	Total beneficiaries	Total benefits (thousands)	Avg. benefits (nominal)	Avg. benefits (real)
MR	Illinois	1,676,914	2,828,644	1,687	605
	Indiana	987,268	1,694,505	1,716	616
	Iowa	503,615	840,728	1,669	599
	Kansas	422,971	728,195	1,722	618
	Michigan	1,600,554	2,795,609	1,747	626
	Minnesota	829,789	1,444,723	1,741	624
	Missouri	936,580	1,525,269	1,629	584
	Nebraska	270,312	453,226	1,677	601
	North Dakota	105,753	170,049	1,608	577
	Ohio	1,689,343	2,740,915	1,622	582
	South Dakota	146,407	234,267	1,600	574
	Wisconsin	976,275	1,662,791	1,703	611
SR	Alabama	760,698	1,233,110	1,621	581
	Arkansas	468,117	732,407	1,565	561
	Delaware	175,408	317,406	1,810	649
	District of Columbia	60,292	99,154	1,645	590
	Florida	3,720,938	6,132,177	1,648	591
	Georgia	1,359,691	2,206,254	1,623	582
	Kentucky	647,855	1,019,674	1,574	565
	Louisiana	583,793	899,530	1,541	553
	Maryland	777,516	1,379,246	1,774	636
	Mississippi	446,981	688,320	1,540	552
	North Carolina	1,611,146	2,674,043	1,660	595
	Oklahoma	561,018	908,245	1,619	581
	South Carolina	883,812	1,482,113	1,677	601
	Tennessee	1,044,660	1,717,632	1,644	590
	Texas	3,149,545	5,124,554	1,627	584
	Virginia	1,174,814	2,018,548	1,718	616
	West Virginia	302,162	487,226	1,612	578
WR	Alaska	81,718	130,347	1,595	572
	Arizona	1,105,267	1,874,684	1,696	608
	California	4,636,107	7,534,273	1,625	583
	Colorado	709,963	1,200,924	1,692	607
	Hawaii	230,841	382,062	1,655	594
	Idaho	282,455	461,090	1,632	586
	Montana	189,757	299,295	1,577	566
	Nevada	438,116	706,383	1,612	578
	New Mexico	326,068	510,837	1,567	562
	Oregon	695,077	1,156,068	1,663	597
	Utah	319,644	549,683	1,720	617
	Washington	1,068,554	1,872,199	1,752	628
	Wyoming	91,386	154,964	1,696	608

Note: *Abbreviations:* NR – Northeast Region; MR – Midwest Region; SR – South Region; WR – West Region; Avg. – Average.

Table 2. COLA and CPI-U NSA determined by each expert

Indicator	e_1	e_2	e_3
COLA 2023	1.9	3.1	3.6
COLA 2024	1.8	2.0	2.5
COLA 2025	1.8	2.5	3.1
CPI-U NSA Dec-2025	315.729	322.914	328.287

Table 3. December 2025 scenario forecasts of the nominal and real average Social Security benefits for retired workers (values in dollars)

	State	Nominal values			Real values		
		S_1	S_2	S_3	S_1	S_2	S_3
NR	Connecticut	2,199	2,245	2,281	696.5	695.3	694.7
	Maine	1,888	1,927	1,958	597.9	596.9	596.4
	Massachusetts	2,085	2,128	2,162	660.3	659.1	658.6
	New Hampshire	2,185	2,230	2,266	691.9	690.7	690.2
	New Jersey	2,201	2,247	2,283	697.0	695.9	695.4
	New York	2,034	2,077	2,110	644.3	643.3	642.9
	Pennsylvania	2,062	2,106	2,139	653.2	652.1	651.6
	Rhode Island	2,056	2,099	2,132	651.2	650.0	649.5
	Vermont	2,041	2,084	2,117	646.4	645.2	644.7
MR	Illinois	2,008	2,050	2,082	635.9	634.8	634.3
	Indiana	2,051	2,095	2,129	649.7	648.8	648.4
	Iowa	2,000	2,042	2,074	633.6	632.4	631.8
	Kansas	2,066	2,108	2,141	654.2	652.9	652.3
	Michigan	2,088	2,133	2,168	661.2	660.5	660.3
	Minnesota	2,109	2,153	2,188	667.9	666.9	666.5
	Missouri	1,951	1,992	2,024	617.8	616.8	616.5
	Nebraska	2,016	2,058	2,090	638.6	637.3	636.7
	North Dakota	1,934	1,973	2,003	612.5	611.0	610.2
	Ohio	1,930	1,971	2,003	611.2	610.4	610.1
	South Dakota	1,934	1,975	2,006	612.7	611.6	611.1
	Wisconsin	2,040	2,083	2,117	646.3	645.2	644.8
SR	Alabama	1,956	1,997	2,029	619.5	618.5	618.1
	Arkansas	1,883	1,923	1,954	596.5	595.6	595.2
	Delaware	2,184	2,230	2,266	691.7	690.6	690.2
	District of Columbia	2,029	2,071	2,103	642.8	641.4	640.7
	Florida	1,973	2,014	2,047	624.8	623.8	623.4
	Georgia	1,955	1,998	2,031	619.4	618.7	618.5
	Kentucky	1,891	1,931	1,962	598.8	597.9	597.6
	Louisiana	1,843	1,882	1,912	583.9	582.8	582.4
	Maryland	2,148	2,192	2,226	680.2	678.8	678.2

End of Table 3

	State	Nominal values			Real values		
		S_1	S_2	S_3	S_1	S_2	S_3
SR	Mississippi	1,857	1,896	1,927	588.1	587.2	586.8
	North Carolina	2,004	2,047	2,081	634.9	634.1	633.8
	Oklahoma	1,946	1,986	2,017	616.2	615.0	614.4
	South Carolina	2,029	2,072	2,105	642.6	641.6	641.2
	Tennessee	1,985	2,027	2,060	628.6	627.8	627.6
	Texas	1,952	1,993	2,024	618.3	617.1	616.6
	Virginia	2,084	2,128	2,162	660.1	659.0	658.5
	West Virginia	1,923	1,962	1,993	609.0	607.6	606.9
WR	Alaska	1,905	1,943	1,973	603.2	601.8	601.1
	Arizona	2,035	2,078	2,112	644.6	643.6	643.3
	California	1,936	1,977	2,009	613.2	612.3	611.9
	Colorado	2,043	2,086	2,120	647.1	646.1	645.7
	Hawaii	1,987	2,029	2,061	629.2	628.2	627.8
	Idaho	1,960	2,002	2,035	620.9	620.0	619.8
	Montana	1,885	1,924	1,955	596.9	595.9	595.5
	Nevada	1,920	1,962	1,993	608.2	607.4	607.2
	New Mexico	1,879	1,918	1,948	595.1	594.0	593.5
	Oregon	1,987	2,029	2,061	629.4	628.3	627.8
	Utah	2,069	2,112	2,145	655.3	654.0	653.3
	Washington	2,104	2,148	2,181	666.3	665.0	664.5
	Wyoming	2,037	2,079	2,111	645.2	643.7	643.0

Note: *Abbreviations:* NR – Northeast Region; MR – Midwest Region; SR – South Region; WR – West Region.

Table 4. December 2025 aggregated results of the real average Social Security benefits for retired workers (values in dollars)

	State	OWA-RAP	AOWA-RAP	IOWA-RAP	OWHA-RAP	OWQA-RAP	POWA-RAP
NR	Connecticut	696.1	695.0	695.3	696.1	696.1	695.7
	Maine	597.6	596.7	596.9	597.6	597.6	597.3
	Massachusetts	659.9	658.8	659.1	659.9	659.9	659.5
	New Hampshire	691.5	690.5	690.7	691.5	691.5	691.1
	New Jersey	696.6	695.7	695.9	696.6	696.6	696.3
	New York	643.9	643.1	643.3	643.9	643.9	643.6
	Pennsylvania	652.8	651.9	652.1	652.8	652.8	652.5
	Rhode Island	650.8	649.8	650.0	650.8	650.8	650.5
	Vermont	646.0	645.0	645.2	646.0	646.0	645.6

End of Table 4

	State	OWA-RAP	AOWA-RAP	IOWA-RAP	OWHA-RAP	OWQA-RAP	POWA-RAP
MR	Illinois	635.5	634.5	634.8	635.5	635.5	635.2
	Indiana	649.4	648.6	648.8	649.4	649.4	649.1
	Iowa	633.2	632.1	632.4	633.2	633.2	632.8
	Kansas	653.8	652.6	652.9	653.8	653.8	653.4
	Michigan	661.0	660.4	660.5	661.0	661.0	660.8
	Minnesota	667.6	666.7	666.9	667.6	667.6	667.3
	Missouri	617.5	616.7	616.9	617.5	617.5	617.2
	Nebraska	638.1	637.0	637.3	638.1	638.1	637.8
	North Dakota	612.0	610.6	611.0	612.0	612.0	611.5
	Ohio	610.9	610.3	610.4	610.9	610.9	610.7
	South Dakota	612.3	611.4	611.6	612.3	612.3	612.0
	Wisconsin	645.9	645.0	645.2	645.9	645.9	645.6
SR	Alabama	619.1	618.3	618.5	619.1	619.1	618.9
	Arkansas	596.2	595.4	595.6	596.2	596.2	595.9
	Delaware	691.4	690.4	690.7	691.4	691.4	691.0
	District of Columbia	642.3	641.0	641.4	642.3	642.3	641.9
	Florida	624.5	623.6	623.8	624.5	624.5	624.2
	Georgia	619.1	618.6	618.7	619.1	619.1	619.0
	Kentucky	598.5	597.8	597.9	598.5	598.5	598.3
	Louisiana	583.5	582.6	582.8	583.5	583.5	583.2
	Maryland	679.7	678.5	678.8	679.7	679.7	679.3
	Mississippi	587.8	587.0	587.2	587.8	587.8	587.5
	North Carolina	634.6	634.0	634.1	634.6	634.6	634.4
	Oklahoma	615.8	614.7	615.0	615.8	615.8	615.4
	South Carolina	642.2	641.4	641.6	642.2	642.2	642.0
	Tennessee	628.3	627.7	627.8	628.3	628.3	628.1
	Texas	617.9	616.9	617.1	617.9	617.9	617.5
	Virginia	659.7	658.7	659.0	659.7	659.7	659.4
	West Virginia	608.5	607.3	607.6	608.5	608.5	608.1
WR	Alaska	602.7	601.4	601.8	602.7	602.7	602.3
	Arizona	644.2	643.5	643.6	644.2	644.2	644.0
	California	612.9	612.1	612.3	612.9	612.9	612.6
	Colorado	646.7	645.9	646.1	646.7	646.7	646.4
	Hawaii	628.9	628.0	628.2	628.9	628.9	628.6
	Idaho	620.6	619.9	620.1	620.6	620.6	620.3
	Montana	596.6	595.7	595.9	596.6	596.6	596.3
	Nevada	608.0	607.3	607.5	608.0	608.0	607.7
	New Mexico	594.7	593.7	594.0	594.7	594.7	594.4
	Oregon	629.0	628.1	628.3	629.0	629.0	628.7
	Utah	654.8	653.7	654.0	654.8	654.8	654.4
	Washington	665.8	664.8	665.1	665.8	665.8	665.5
	Wyoming	644.7	643.3	643.7	644.6	644.7	644.2

Note: *Abbreviations:* NR – Northeast Region; MR – Midwest Region; SR – South Region; WR – West Region.

Table 5. Comparison between nominal and real growth

	State	Nominal values			Real values		
		Dec-2021	Dec-2025 IOWA	Growth	Dec-2021	Dec-2025 IOWA-RAP	Growth
NR	Connecticut	1,833	2,248	23%	658	695	6%
	Maine	1,569	1,930	23%	563	597	6%
	Massachusetts	1,729	2,131	23%	620	659	6%
	New Hampshire	1,802	2,233	24%	646	691	7%
	New Jersey	1,835	2,250	23%	658	696	6%
	New York	1,708	2,080	22%	612	643	5%
	Pennsylvania	1,720	2,108	23%	617	652	6%
	Rhode Island	1,708	2,101	23%	612	650	6%
	Vermont	1,694	2,086	23%	608	645	6%
MR	Illinois	1,687	2,052	22%	605	635	5%
	Indiana	1,716	2,097	22%	616	649	5%
	Iowa	1,669	2,044	22%	599	632	6%
	Kansas	1,722	2,111	23%	618	653	6%
	Michigan	1,747	2,135	22%	626	661	5%
	Minnesota	1,741	2,156	24%	624	667	7%
	Missouri	1,629	1,994	22%	584	617	6%
	Nebraska	1,677	2,060	23%	601	637	6%
	North Dakota	1,608	1,975	23%	577	611	6%
	Ohio	1,622	1,973	22%	582	610	5%
	South Dakota	1,600	1,977	24%	574	612	7%
	Wisconsin	1,703	2,086	22%	611	645	6%
SR	Alabama	1,621	1,999	23%	581	619	6%
	Arkansas	1,565	1,925	23%	561	596	6%
	Delaware	1,810	2,233	23%	649	691	6%
	District of Columbia	1,645	2,073	26%	590	641	9%
	Florida	1,648	2,017	22%	591	624	6%
	Georgia	1,623	2,000	23%	582	619	6%
	Kentucky	1,574	1,933	23%	565	598	6%
	Louisiana	1,541	1,884	22%	553	583	5%
	Maryland	1,774	2,194	24%	636	679	7%
	Mississippi	1,540	1,898	23%	552	587	6%
	North Carolina	1,660	2,050	24%	595	634	7%
	Oklahoma	1,619	1,988	23%	581	615	6%
	South Carolina	1,677	2,074	24%	601	642	7%
	Tennessee	1,644	2,030	23%	590	628	6%
	Texas	1,627	1,995	23%	584	617	6%
	Virginia	1,718	2,130	24%	616	659	7%
	West Virginia	1,612	1,964	22%	578	608	5%

End of Table 5

	State	Nominal values			Real values		
		Dec-2021	Dec-2025 IOWA	Growth	Dec-2021	Dec-2025 IOWA-RAP	Growth
WR	Alaska	1,595	1,945	22%	572	602	5%
	Arizona	1,696	2,081	23%	608	644	6%
	California	1,625	1,979	22%	583	612	5%
	Colorado	1,692	2,089	23%	607	646	6%
	Hawaii	1,655	2,031	23%	594	628	6%
	Idaho	1,632	2,004	23%	586	620	6%
	Montana	1,577	1,926	22%	566	596	5%
	Nevada	1,612	1,964	22%	578	607	5%
	New Mexico	1,567	1,920	23%	562	594	6%
	Oregon	1,663	2,031	22%	597	628	5%
	Utah	1,720	2,114	23%	617	654	6%
	Washington	1,752	2,150	23%	628	665	6%
	Wyoming	1,696	2,081	23%	608	644	6%

Note: *Abbreviations:* NR – Northeast Region; MR – Midwest Region; SR – South Region; WR – West Region.

In Table 4, we can see that New Jersey, Connecticut, and New Hampshire are the three states of the U.S. with the highest results. By contrast, Louisiana, Mississippi, and New Mexico have obtained the lowest amounts. Moreover, the differences between these states are quite significant. For example, the gap between New Jersey and Louisiana is 113.1 dollars for the OWARAP operator.

Likewise, if we look at the results on a regional level, we observe that, on average, the Northeast Region has the highest estimated real average Social Security benefit for retired workers, compared to the South Region, which has the smallest one.

Furthermore, it is interesting to analyze the effect of the inflation adjustment on the outcomes. For example, by looking at Table 5, we can see that in North Carolina, the average Social Security benefit for retired workers in real prices (based on the IOWARAP operator) is expected to increase by 7%. However, if we conduct the same calculations without considering inflation, then it is estimated to increase by 24%. In this case, the real growth is not in line with the current growth, which translates to a considerable loss in the purchasing power of the future beneficiaries of this state. This demonstrates the importance of having information regarding retirement benefits in real prices.

Additionally, Table 6 compares the proposed approach with other statistical forecasting methods. Specifically, with the linear trend (LT) and the double moving average (DMA) of length 2. The state of North Carolina and a 3-period hold-out is considered for the comparative analysis. The performance criteria utilized are the mean absolute error (MAE), the mean squared error (MSE), and the mean absolute percentage error (MAPE). As can be seen, the prediction accuracy is higher with the OWARAP operator. Also, in contrast to the LT and the DMA, the OWARAP operator allows the aggregation of the opinions of individual experts according to the degree of orness. This feature is particularly advantageous when the environment is uncertain.

Note that, for the $OWARAP_1$, it is assumed that all three experts provided entirely correct information regarding the CPI-U NSA and COLA evolution. For the $OWARAP_2$, rational assumptions are made. Concretely, Expert 1 expects an annual growth of 5% of the December CPI-U NSA, Expert 2 of 3%, and Expert 3 of 1%. A similar evolution of the COLA is considered. Lastly, the weighting vector contemplated is W = (0.5, 0.3, 0.2) for both operators.

Table 6. Comparison between forecasting methods

		Actual value	LT	DMA	$OWARAP_1$	$OWARAP_2$
Real avg. benefit	Dec-2019	585	591	590	585	581
	Dec-2020	593	597	599	592	588
	Dec-2021	595	604	609	593	597
Error measure	MAE	0	6	8	1	3
	MSE	0	39	78	2	14
	MAPE	0.0%	1.0%	1.3%	0.2%	0.6%

5. Conclusions

The OWARAP operator is an aggregation operator used for calculating the future average retirement benefit adjusted for inflation. The OWARAP operator is based on the OWA operator. Thus, it provides a parametrized family of aggregation operators ranging from the minimum to the maximum real average retirement benefit. The OWARAP operator can be extended by using order-inducing variables, generalized means, and also probabilities. In the first case, the IOWARAP operator is obtained; in the second case, the GOWARAP operator; and in the last case, the POWARAP operator.

This paper also develops a multi-expert analysis of the use of the OWARAP operator and its extensions in calculating the future average Social Security benefit adjusted for inflation of a retired worker in each state of the U.S. This analysis shows that with the use of these new operators, it is possible to underestimate or overestimate the results according to the attitudinal character of the analyst as well as its preferences. Furthermore, it demonstrates the importance of removing the effect of price inflation in order to obtain a true picture of the future average Social Security benefits for retired workers. By using the new approach, individuals can plan their retirement more properly and thereby maintain their standard of living. Similarly, it can help policy makers make good decisions on retirement-related matters.

In order to continue developing this idea, in future research, it is proposed to study further extensions of the OWARAP, IOWARAP, GOWARAP, and POWARAP operators. Also, apply these aggregation operators in other countries, such as France or Canada. Lastly, it is suggested to develop new algorithms for forecasting retirement indicators.

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