



GENERALIZED DEMATEL TECHNIQUE WITH CENTRALITY MEASUREMENTS

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Abstract. DEMATEL technique is a useful tool for understanding the influential relationship between criteria of a systematic problematic in structural modeling, and has received much attention in the field of decision analysis recently. However, the past papers focused on the applications of DEMATEL technique and ignored the convergence problem of the approach. In addition, two simple indicators, i.e., in-degree and out-degree centralities, used in DEMATEL technique cannot fully represent the insight of the network relationship. In this paper, we propose a general DEMATEL technique which incorporated the concept based on interaction diminishing effect. The traditional DEMATEL technique can be considered as a special case of the proposed method when we ignore the effect. Later, we give six important indicators which can be used in DEMATEL technique to conclude the relative importance of criteria. In addition, a numerical example is used to demonstrate the proposed method and the applications of the indicators.

Keywords: decision analysis, DEMATEL technique, influential relationship, systematic problematic, structural modeling, indicators.

JEL classification: C67.

Introduction

Structural modeling is still an attractive and visual tool for exploring the influential relationship between criteria (or called aspects, factors, attributes, or elements), although it has already been developed for half a century. For example, the decision making trial and evaluation laboratory (DEMATEL) technique was first developed in the 1970s (Fontela, Gabus 1974, 1976; Warfield 1976). According to the Google Scholar statistics, there are over 350 academic papers using DEMATEL in their titles since 2005, which justify the practical applications of DEMATEL.

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The concept of DEMATEL technique comes from the well-known Leontief's input and output model (Leontief 1949), which has been widely used in economics. The major advantage of DEMATEL technique is that it can obtain the structural model of a systematic problematique through investigating the strength of binary relations (pairwise comparison) between entries.

Compared to the past papers, this paper focuses on the issues of the convergence problem and indicator development in DEMATEL technique. The convergence problem of DEMATEL technique can be considered as the violation of Hawkins-Simon condition or strictly diagonally dominant matrices (Bidard 2007). In this situation, DEMATEL technique cannot obtain a convergent solution and, therefore, we will develop a modified DEMATEL technique to deal with this issue here. On the other hand, the conventional DEMATEL technique only uses the sums of rows and columns of total relation matrix to give decision-makers information about the relative importance of entries. In this paper, we propose six importance indicators of criteria for decision-makers to understand the insight of the problematique.

The remaining sections are organized as follows. In Section 2, the contents of DEMATEL technique are outlined. We discuss the convergence problem of DEMATEL technique in Section 3. In Section 4, a modified DEMATEL and centrality measurements of criteria are proposed. We will give a numerical example and compare the result with the conventional DEMATEL technique in Section 5. Finally, we present the discussion and conclusion in the last section.

1. Descriptions of DEMATEL technique

The DEMATEL technique, developed by the Science and Human Affairs Program of the Battelle Memorial Institute of Geneva between 1972 and 1976, is used for researching and solving complicated and intertwined problem groups (Fontela, Gabus 1974, 1976; Warfield 1976). DEMATEL was developed in the belief that pioneering and appropriate use of scientific research methods could improve understanding of a specific problematique, a cluster of intertwined problems, and contribute to identification of workable solutions by a hierarchical structure.

The methodology, according to the concrete characteristics of objective affairs, can confirm the interdependence between entries and restrict the relationship that reflects the characteristic with an essential system and development trend (Hori, Shimizu 1999). Using the DEMATEL method to size and process individual subjective perceptions, brief and impressionistic human insights into problem complexity can be gained. Following the DEMATEL process, the result of the analysis is a visual representation, an individual map of the mind, according to which one can organize one's own actions in the world, if one is to keep internally coherent to respect one's implicit priorities and to reach one's secret goals. Therefore, using DEMATEL technique provides an easy way for researchers to study the structure of a systematic problematique for various issues in the real world.

For example, these papers are distributed over evaluating intertwined effects in investment project (Altuntas, Dereli 2012, 2015), e-learning programs (Tzeng *et al.* 2007),

customers' choice behavior model (Hsu et al. 2007), environment assessment (Tseng, Lin 2009; Chen et al. 2009, 2010), service-industry evaluation (Chen et al. 2011; Cheng et al. 2012; Liu et al. 2013), quality control (Dey et al. 2012; Tsai, Hsu 2010; Wang et al. 2014), and risk evaluation (Chang, Cheng 2011; Ou Yang et al. 2009, 2013). On the other hand, many papers have extended DEMATEL technique to consider fuzzy environments (Altuntas et al. 2014; Fekri et al. 2009; Jeng, Tzeng 2011; Lin, Wu 2004; Hsu et al. 2007; Hu et al. 2012; Lu et al. 2013; Shen, Tzeng 2014) and stochastic DEMATEL (Tamura, Akazawa 2005; Tamura et al. 2006).

The original concept of DEMATEL technique comes from Leontief's input and output model (Leontief 1949), which describes the processes of how one sector distributes its output to the other sectors of the economy (Tzeng, Tsaur 1995). The outline of the DEMATEL technique can be described as follows. First, respondents are asked to indicate the binary relation and the strength of each relation from 0 to 4; where measuring scale from 0 to 4 can be presented by nature language, as (0) represents no influence whatsoever, (1) represents low influence, (2) represents medium influence, (3) represents high influence, and (4) represents extremely high influence (Peng, Tzeng 2013). A higher score indicates that the respondent has expressed that factor i exerts a stronger possible direct influence on the inability of factor j , or, in positive terms, that greater improvement of i is required to improve j . Then, the normalized direct influence matrix D can be obtained by normalizing the direct influence matrix $X = [x_{ij}]$, in which all principal diagonal elements are equal to zero such that

$$D = s \cdot X, s > 0, \tag{1}$$

where

$$s = \min_{i,j} \left\{ 1 / \max_{1 \leq i \leq n} \sum_{j=1}^n |x_{ij}|, 1 / \max_{1 \leq j \leq n} \sum_{i=1}^n |x_{ij}| \right\}. \tag{2}$$

Then, the total relation matrix F can be obtained by

$$F = \sum_{i=1}^{\infty} D^i = D(I - D)^{-1}, \tag{3}$$

where I denotes the identity matrix.

[Explanation]:

$$F = \sum_{i=1}^h D^i = D + D^2 + D^3 + \dots + D^h = D(I + D + D^2 + \dots + D^{h-1}) [(I - D)(I - D)^{-1}] =$$

$$D(I - D^h)(I - D)^{-1}, \text{ then,}$$

$$F = D(I - D)^{-1}, \text{ when } \lim_{h \rightarrow \infty} D^h = [0]_{n \times n},$$

where $D = [d_{ij}]_{n \times n}$, $0 \leq d_{ij} < 1$, $0 < \sum_{j=1}^n d_{ij} \leq 1$, $0 < \sum_{i=1}^n d_{ij} \leq 1$. If at least one row or col-

umn of summation is equal to 1 (but not all) in $\sum_{j=1}^n d_{ij}$ and $\sum_{i=1}^n d_{ij}$, then we can guarantee $\lim_{h \rightarrow \infty} D^h = [0]_{n \times n}$.

Furthermore, decision-makers can depict the network relationship diagram according to the result of each row and column sums. However, we ignore the succeeding steps of DEMATEL and just focus on the purpose here. Readers can refer to (Tzeng, Huang 2011) to understand the content of DEMATEL technique in detail.

Recently, many papers considered using DEMATEL to solve various decision problems. For example, Shieh *et al.* (2010) used DEMATEL in identifying key success factors of hospital service quality; Tzeng and Lin (2009) developed a cause and effect model of municipal solid waste management in Metro Manila by fuzzy DEMATEL; Chang and Cheng (2011) employed DEMATEL to evaluating the risk of failure; Bai and Sarkis (2013) combined grey and DEMATEL to evaluate business process management critical success factors. These papers also justify the reason of a general DEMATEL which can consider more complex decision problems.

2. Convergence condition and problem of DEMATEL

Since the concept of DEMATEL technique comes from Leontief’s input and output analysis (I-O analysis), it shares the same conditions that the matrix $(I - D)^{-1}$ (total output vector $x = F(I - D)^{-1}$ or written $x' = (I - D)^{-1}F'$, where the superscript ' denotes the transpose operation, and F denotes the final demand in I-O analysis) should be existent and positive (Tzeng, Tsaur 1995). Let us consider the following theorems and definitions to describe the issues of invariability and nonnegative inverse in DEMATEL technique.

Definition 1. If A is a real $n \times n$ matrix and can be written in the form $A = sI - B$ where $B \geq 0$ and $s > \rho(B)$, then A is called a nonsingular M -matrix (Frobenius 1912), where $\rho(\cdot)$ denotes the spectral radius.

Definition 2. If A is a real $n \times n$ matrix, it is said to be inverse positive if $A^{-1} \geq 0$. If $A^{-1} > 0$, then A is said to be strictly inverse positive.

Definition 3. A real $n \times n$ matrix A is said to be reducible if there exists a permutation matrix P such that

$$C = PAP' = \begin{bmatrix} A_{11} & A_{12} \\ \mathbf{0} & A_{22} \end{bmatrix}, \tag{4}$$

where $A_{11} \in R^{r \times r}$, $A_{22} \in R^{(n-r) \times (n-r)}$, and $A_{12} \in R^{r \times (n-r)}$, $\forall r \in (0, n)$.

Definition 4. A real $n \times n$ matrix A is said to be irreducible if it is not reducible.

Definition 5. A directed graph G is strongly connected if for any ordered pair (i, j) of vertices of G , there exists a sequence of edges which leads from i to j .

Let a_{ij} be the entry of a real $n \times n$ matrix A . **Definition 5** indicates that if there exists a sequence of q edges from i to j , $a_{ij}^{(q)} > 0$, it translates into **Theorem 1**.

Theorem 1. A real $n \times n$ matrix A is said to be irreducible iff $G(A)$ is strongly connected in graph.

Theorem 2. Let A be a real $n \times n$ nonsingular M -matrix. Then the following properties are equivalent:

- (i) A^{-1} is nonnegative.
- (ii) The principal minors of A are strictly positive (called the Hawkins-Simon condition).
- (iii) If A is irreducible, then A is strictly inverse positive.

Clearly, the matrix $(I - D)$ in DEMATEL is a special case of an M -matrix, where $s = 1$. From **Theorem 2**, it can be seen that if the matrix $(I - D)$ in DEMATEL is a nonsingular M -matrix, we can ensure that the matrix $(I - D)^{-1} \geq 0$ exists, no matter if it is reducible or irreducible. Usually, the well-known Hawkins-Simon condition is used to ensure that $(I - D)$ is a nonsingular M -matrix. However, it is cumbersome in practice when dealing with complex problems, since it needs to calculate each principal minor of the matrix. Note that readers who are interested in the characteristics of the Hawkins-Simon condition can refer to Bidard (2007) for a more detailed description. In addition, if the Hawkins-Simon condition is not satisfied, we do not even know how to modify the matrix, X or D , in DEMATEL. Hence, instead of considering the Hawkins-Simon condition here, we introduce the method of strictly diagonally dominant matrices as follows.

Definition 6. A real $n \times n$ matrix, A , is a strictly diagonally dominant matrix if the magnitude of the diagonal element in a row/column is larger than the sum of the magnitudes of all the other (non-diagonal) entries in that row/column.

Theorem 3. The Hawkins-Simon condition is equivalent to the condition of strictly diagonally dominant matrices.

Theorem 3 claims that if the matrix $(I - D)$ is diagonally dominant, that is,

$$\sum_{j \neq i} |a_{ij}| < |a_{ii}|, i = 1, \dots, n \text{ or } \sum_{i \neq j} |a_{ij}| < |a_{jj}|, j = 1, \dots, n \tag{5}$$

or, if the matrix $(I - D)$ is irreducible and

$$\sum_{j \neq i} |a_{ij}| \leq |a_{ii}|, i = 1, \dots, n \text{ or } \sum_{i \neq j} |a_{ij}| \leq |a_{jj}|, j = 1, \dots, n. \tag{6}$$

With strict inequality for at least one i or j , then $(I - D)^{-1}$ can be ensured to be positive and existent.

Proof. Let A be diagonally dominant, the diagonal matrix of A is represented as $diag[a_{11}, \dots, a_{nn}]$ and define $B = I - D^{-1}A$. Then, if A is a diagonally dominant matrix, it implies that

$$\sum_{j \neq i} |b_{ij}| < 1, i = 1, \dots, n, \tag{7}$$

and

$$\rho(B) < 1, \tag{8}$$

according to Gershgorin's circle theorem (Gershgorin 1931).

Since $D^{-1}A = I - B$ and $\rho(B) < 1$, $D^{-1}A$ is an M -matrix, consequently A is an M -matrix.

Definition 7. If A is a real $n \times n$ matrix and can be written in the form $A = sI - B$ where $B \geq 0$ and $s = \rho(B)$, then A is a singular matrix.

According to the above definitions, we can conclude that the convergence problem of DEMATEL can only happen when the normalized direct relation matrix is a stochastic matrix, i.e., $\rho(\mathbf{B}) = 1$, if the self-loop effect is ignored. In this situation, $(\mathbf{I} - \mathbf{D})^{-1}$ will go to infinity and result in the convergence problem. To overcome the above problem, we can develop a modified DEMATEL technique to obtain the approximate solution as the following section.

3. Modified DEMATEL and centrality measurements

In this paper, we consider to extend DEMATEL technique to be workable in any situations. Lee *et al.* (2013) provided a revised DEMATEL technique to solve the convergence problem by replacing the normalizing parameter s as:

$$s = \min_{i,j} \left\{ 1 / \max_{1 \leq i \leq n} \sum_{j=1}^n |x_{ij}|, 1 / \left(\varepsilon + \max_{1 \leq j \leq n} \sum_{i=1}^n |x_{ij}| \right) \right\}, \tag{9}$$

where ε denotes a very small positive number. Then, other steps are similar to the traditional DEMATEL technique. Note that Eq. (9) was proposed by Lee *et al.* (2013) to solve the convergence problem of DEMATEL. It is an approximate formula of Eq. (2). However, the proposed method can ensure the convergence of DEMATEL. Hence, we use Eq. (2) here to normalize the direct influence matrix. In contrast to the above method, the proposed method can derive a closed-form solution and provides more flexible way to apply DEMATEL technique in more complex situations.

The main change of the proposed method is the process of calculating total relation matrix. Here, \mathbf{D} is normalized direct relation matrix and can be viewed as the first-order weighted interaction matrix. The second and other high-order weighted interaction matrix can calculate its corresponding indirect relation matrix $\mathbf{D}^2, \mathbf{D}^3, \dots$ and so on. In addition, we should also consider the interaction diminishing effect. Note that the interaction diminishing effect can be considered as the control parameter of DEMATEL technique to weight the level of interaction effect. The larger of the interaction diminishing effect, the larger the influence of the interaction effect.

Hence, the total relation matrix, \mathbf{F} , here can be calculated by summing $\mathbf{D}, (1 - \alpha) \mathbf{D}^2, \dots$ as:

$$\mathbf{F} = \lim_{h \rightarrow \infty} [\mathbf{D} + (1 - \alpha) \mathbf{D}^2 + \dots + (1 - \alpha)^{h-1} \mathbf{D}^h] = \mathbf{D}[\mathbf{I} - (1 - \alpha) \mathbf{D}]^{-1},$$

$$\text{when } \lim_{h \rightarrow \infty} \mathbf{D}^h = [\mathbf{0}]_{n \times n}, \tag{10}$$

where $\alpha \in [0,1]$ denotes the interaction diminishing effect which determines the level of the decreasing velocity of the interaction effect.

Note that when $\alpha = 0$, Eq. (8) reduces to the original DEMATEL. On the other hand, when $\alpha = 1$, only the direct relation matrix is considered. The degree of alpha indicates the importance of the interaction effect between criteria. In this paper, we adopt $\alpha = 0.2$ to deal with the interaction effect for only the reason of demonstration. However, a decision-maker can determine an appropriate alpha value according to the strength of the interaction ef-

fect. For example, if we consider to calculate the maximum interaction effect, we can set $\alpha = 0$; the medium interaction effect, we can set $\alpha = 0.5$; the strong interaction, we can set $\alpha = 0.2$, and so on.

The convergence problem of DEMATEL technique happens when $(I - D)^{-1}$ is singular and $\alpha = 0$, according to Eq. (10). Hence, a simple way to solve the above problem is to set α as a small positive number to obtain an approximate solution of DEMATEL. In addition, the result of the proposed model is convergent iff $Z = \Lambda[I - (1 - \alpha) \Lambda]^{-1}, \forall \alpha \in (0,1]$ in generalized DEMATEL technique.

Proof:

Case 1. If Λ is a nonsingular matrix

The determinant of $(1 - \alpha) \Lambda, \forall \alpha \in [0,1]$ is less than 1. Hence, the determinant of $[I - (1 - \alpha) \Lambda], \forall \alpha \in [0,1]$, is larger than zero and ensures $Z = \Lambda[I - (1 - \alpha) \Lambda]^{-1} = C$, where C is a constant matrix.

Case 2. If Λ is a singular matrix

Since Λ is a singular matrix, the determinant of $(1 - \alpha) \Lambda, \forall \alpha \in [0,1]$ is less than 1. Hence, the determinant of $\Lambda[I - (1 - \alpha) \Lambda], \forall \alpha \in [0,1]$, is larger than zero and ensures $Z = \Lambda[I - (1 - \alpha) \Lambda]^{-1} = C$, where C is a constant matrix.

The above results show the convergence problem of DEMATEL can be solved by the proposed method iff $\alpha \in (0,1]$.

On the other hand, in traditional DEMATEL technique, the sums of rows and columns of total relation matrix are the only two indicators to understand the relative importance/influence of entities in a problematique. Here, we provide the following indicators for decision-makers to increase the insight of the network.

Definition 8. A network can be represented by a graph (N, G) , which consists of a set of nodes $N = \{1, \dots, n\}$ and an $n \times n$ matrix $G = [g_{ij}], i, j \in N$ (referred to as an adjacency matrix), where $g_{ij} \in \{0,1\}$ represents the availability of an edge from node i to node j .

1. *In-degree* and *out-degree centralities*: *in-degree* is the number of incoming edges, i.e., the column sums of the total relation matrix, and *out-degree* is the number of outgoing ones, i.e., the row sums of the total relation matrix. Hence, node i 's in-degree and out-degree centralities can be respectively defined as

$$C_{id}(n_i) = \frac{\sum_j \omega_{ji} g_{ji}}{\omega^* |N|}, \tag{11}$$

and

$$C_{od}(n_i) = \frac{\sum_j \omega_{ij} g_{ij}}{\omega^* |N|}, \tag{12}$$

where ω_{ij} denotes the link weights, i.e., the i th row and j th column of the total relation matrix, ω^* is the maximum value of all link values in the network and $|N|$ denotes the maximum linked number of the network.

2. *O-I index*: *O-I index* measures the net influence of a node in the network. The out-degree of Node i indicates the influence of Node i to all other nodes; the in-degree

of Node i indicates the influence of Node i from all other nodes. Hence, *O-I index* of Node i is defined as:

$$O-I\ index(n_i) = \frac{(C_{od}(n_i) - C_{id}(n_i))}{(C_{id}(n_i) + C_{od}(n_i))} \in [0,1]. \tag{13}$$

3. *Closeness centrality*: *closeness* is based on the inverse of the distance of each node to every other node in the network. A node is probably considered important if it is relatively close to all other nodes. The *closeness centrality* can be defined by:

$$C_c(n_i) = \frac{\omega^*(|N|-1)}{\sum_{j \neq i} \omega_{ij} d(i, j)}, \tag{14}$$

where $d(i, j)$ denotes distance between nodes i and j .

4. *Betweenness centrality*: *betweenness* measures the degree to which a given node lies on the shortest paths (geodesics) between other nodes in the network. That is, it counts the number of shortest paths between i and k that node j resides on. High betweenness nodes connect different groups and have control over the flow of information in a network. The *betweenness centrality* can be defined as:

$$C_b(n_i) = \frac{\sum_{j < k} \omega_{ij} g_{jk}(n_i) / g_{jk}}{\omega^*(|N|^2 + |N| + 2)}, \tag{15}$$

where g_{jk} denotes the number of geodesics connecting jk , and $g_{jk}(n_i)$ is the number that node i is on. The denominator of Eq. (15) indicates all possible combinations of two nodes excluding the center of the stars (Freeman 1977, 1979).

5. *Eigenvector centrality*: Eigenvector centrality is often connected with the notion of power or the idea that a node is important if it is connected to other important nodes. While other centralities reward all links equally, eigenvector centrality makes differences by including also the links of the neighbors and of the neighbors of the neighbors etc. The *eigenvector centrality* of node i can be defined as:

$$C_e(n_i) = \frac{c_i}{\sqrt{0.5}},$$

$$\lambda c_e = W c_e, \tag{16}$$

$$c_e = [c_1, \dots, c_n]',$$

where λ is the largest eigenvalue of the adjacency matrix W and c_e is the corresponding eigenvector. Independently from the network size, the maximum value of denominator of Eq. (16) is always $\sqrt{0.5}$ (Bonacich 1972).

Next, we give a numerical example to demonstrate the proposed method and further explanation for the problematique according to the proposed indicators.

4. A numerical example

Quality control is a process by which entities review the quality of all factors involved in production. In project management, quality control requires the project manager and the

project team to inspect the accomplished work to ensure its alignment with the project scope. In this example, six critical factors (S_1 to S_6) are considered in the project management, such as: (1) agreeing on setting the project goals, (2) developing clearly defined plans with assigned responsibilities and accountabilities, (3) managing the project scope effectively, (4) cultivating constant effective communications, (5) making sure you have management support, (6) expanding the competence sets for enhancing the ability in implementation. The decision-maker wants to understand the relationship and relative importance between critical factors to appropriately allocate and distribute resource.

Let the initial direct influence matrix be quantified by an expert as:

$$X = \begin{bmatrix} 1 & 2 & 1 & 1 & 0 & 4 \\ 0 & 3 & 0 & 3 & 0 & 2 \\ 4 & 2 & 0 & 3 & 3 & 1 \\ 3 & 3 & 0 & 3 & 0 & 3 \\ 2 & 2 & 2 & 4 & 2 & 3 \\ 3 & 4 & 2 & 4 & 2 & 2 \end{bmatrix}.$$

First, we normalize the above direct-influence matrix as:

$$D = \begin{bmatrix} 0.0556 & 0.1111 & 0.0556 & 0.0556 & 0.0000 & 0.2222 \\ 0.0000 & 0.1667 & 0.0000 & 0.1667 & 0.0000 & 0.1111 \\ 0.2222 & 0.1111 & 0.0000 & 0.1667 & 0.1667 & 0.0556 \\ 0.1667 & 0.1667 & 0.0000 & 0.1667 & 0.0000 & 0.1667 \\ 0.1111 & 0.1111 & 0.1111 & 0.2222 & 0.1111 & 0.1667 \\ 0.1667 & 0.2222 & 0.1111 & 0.2222 & 0.1111 & 0.1111 \end{bmatrix}.$$

Here, we use the proposed method and respectively let $\alpha = 0$, $\alpha = 0.1$, $\alpha = 0.2$, and $\alpha = 0.3$ here to calculate the full direct/indirect influence matrix F of DEMATEL as:

$$F_0 = \begin{bmatrix} 0.2308 & 0.3713 & 0.1255 & 0.3186 & 0.0781 & 0.4363 \\ 0.1346 & 0.3839 & 0.0446 & 0.3838 & 0.0446 & 0.2898 \\ 0.4423 & 0.4394 & 0.0943 & 0.5036 & 0.2521 & 0.3756 \\ 0.3654 & 0.4822 & 0.0777 & 0.4740 & 0.0703 & 0.4460 \\ 0.4038 & 0.5178 & 0.2184 & 0.6445 & 0.2324 & 0.5312 \\ 0.4615 & 0.6558 & 0.2182 & 0.6677 & 0.2289 & 0.5041 \end{bmatrix};$$

$$F_{0.1} = \begin{bmatrix} 0.1881 & 0.3074 & 0.1092 & 0.2539 & 0.0606 & 0.3813 \\ 0.1016 & 0.3321 & 0.0329 & 0.3320 & 0.0328 & 0.2455 \\ 0.3869 & 0.3564 & 0.0724 & 0.4192 & 0.2305 & 0.2996 \\ 0.3159 & 0.4056 & 0.0589 & 0.3979 & 0.0519 & 0.3787 \\ 0.3337 & 0.4155 & 0.1923 & 0.5400 & 0.2044 & 0.4403 \\ 0.3894 & 0.5478 & 0.1914 & 0.5580 & 0.2007 & 0.4079 \end{bmatrix};$$

$$F_{0.2} = \begin{bmatrix} 0.1573 & 0.2613 & 0.0972 & 0.2073 & 0.0476 & 0.3421 \\ 0.0777 & 0.2944 & 0.0246 & 0.2942 & 0.0244 & 0.2137 \\ 0.3472 & 0.2969 & 0.0563 & 0.3587 & 0.2150 & 0.2444 \\ 0.2804 & 0.3501 & 0.0452 & 0.3431 & 0.0388 & 0.3299 \\ 0.2828 & 0.3423 & 0.1734 & 0.4649 & 0.1839 & 0.3749 \\ 0.3373 & 0.4703 & 0.1722 & 0.4789 & 0.1802 & 0.3385 \end{bmatrix};$$

and

$$F_{0.3} = \begin{bmatrix} 0.1339 & 0.2265 & 0.0880 & 0.1720 & 0.0375 & 0.3129 \\ 0.0597 & 0.2656 & 0.0185 & 0.2655 & 0.0183 & 0.1896 \\ 0.3173 & 0.2525 & 0.0439 & 0.3132 & 0.2033 & 0.2023 \\ 0.2536 & 0.3081 & 0.0349 & 0.3018 & 0.0291 & 0.2928 \\ 0.2440 & 0.2874 & 0.1590 & 0.4082 & 0.1681 & 0.3257 \\ 0.2979 & 0.4119 & 0.1577 & 0.4193 & 0.1646 & 0.2860 \end{bmatrix}.$$

The degree of alpha indicates the importance of the interaction effect between criteria. In this paper, we adopt $\alpha = 0.2$ to deal with the interaction effect for only the reason of demonstration. However, a decision-maker can determine an appropriate alpha value according to the strength of the interaction effect. For example, if we consider to calculate the maximum interaction effect, we can set $\alpha = 0$; the medium interaction effect, we can set $\alpha = 0.5$; the strong interaction, we can set $\alpha = 0.2$, and so on.

Next, we can calculate the following indicators, which are mentioned previous, of $F_{0.2}$ to understand the insight of the network as shown in Table 1, according to Eqs. (11)–(16), respectively.

Table 1. Full direct/indirect influence matrix with various alpha

$F_{0.2}$	<i>In-degree Centrality</i> Eq. (11)	<i>Out-degree Centrality</i> Eq. (12)	<i>O-I index</i> Eq. (13)	<i>Closeness Centrality</i> Eq. (14)	<i>Betweenness Centrality</i> Eq. (15)	<i>Eigenvector Centrality</i> Eq. (16)
S_1	0.516	0.387	-0.143	0.128	0.050	0.515
S_2	0.701	0.323	-0.369	0.195*	0.000	0.583
S_3	0.198	0.528	0.455*	0.083	0.200*	0.500
S_4	0.747*	0.483	-0.215	0.122	0.000	0.640*
S_5	0.240	0.634	0.451	0.074	0.000	0.575
S_6	0.642	0.688*	0.035	0.074	0.000	0.636

For example, the In-degree Centrality of S_1 is calculated by Eq. (11) as $(0.1573 + 0.0777 + \dots + 0.3373) \div (\max\{0.1573, 0.2613, \dots, 0.3385\} \times 6) = 0.516$. On the other hand, the Out-degree Centrality of S_1 is calculated by Eq. (12) as $(0.1573 + 0.2613 + \dots + 0.3421) \div (\max\{0.1573, 0.2613, \dots, 0.3385\} \times 6) = 0.387$. Other indicators can also be derived step by step using the similar way above.

The traditional DEMATEL technique only uses the information of the sum of rows and the sum of columns of the total relation matrix to the relative influence/importance of entities. Note that the conventional DEMATEL uses unscaled *in-degree* and *out-degree centralities* and the proposed methods scale the above two indicators. That is, their results are different. In contrast, the proposed method can derive more insightful indicators to understand the problem of the network from the result of Table 1.

For example, *O-I index* indicates S_3 is influence/important node since it has the most net-influence than others. In addition, S_3 is a critical intermediate factor (i.e., bridge), because many nodes communicate via it, according to *betweenness centrality*. Furthermore, S_2 is also considered as an important node since it is close to every node. It can be realized S_2 as the common factor. Finally, S_4 should be also considered as a critical node since it connects many important nodes based on the information of *eigenvector centrality*.

Discussion and conclusions

The applicability of DEMATEL is widespread ranging from industrial planning and decision making to city planning and design, regional environmental assessment, analyzing global world problematique, and so on. A weighted structural network will be given by analyzing quantitative data on the strength of binary relations on every two criteria. However, the issue of the convergence problem and indicator development seems to be ignored in the papers.

We can give an example to demonstrate the convergence problem of DEMATEL as follows. Let the initial direct influence matrix be represented as:

$$X = \begin{bmatrix} 0 & 0 & 4 & 2 & 0 & 4 \\ 0 & 3 & 0 & 0 & 4 & 0 \\ 4 & 0 & 0 & 4 & 0 & 2 \\ 3 & 0 & 3 & 0 & 0 & 4 \\ 0 & 4 & 0 & 0 & 3 & 0 \\ 3 & 0 & 3 & 4 & 0 & 0 \end{bmatrix}.$$

Then, the normalization of the direct-influence matrix is calculated as:

$$D = \begin{bmatrix} 0 & 0 & 0.4 & 0.2 & 0 & 0.4 \\ 0 & 0.3 & 0 & 0 & 0.4 & 0 \\ 0.4 & 0 & 0 & 0.4 & 0 & 0.2 \\ 0.3 & 0 & 0.3 & 0 & 0 & 0.4 \\ 0 & 0.4 & 0 & 0 & 0.3 & 0 \\ 0.3 & 0 & 0.3 & 0.4 & 0 & 0 \end{bmatrix}.$$

Clearly, the above matrix violates the Hawkins-Simon condition and suffers the convergence problem. That is, F goes to infinity. In this situation, we can set α as a small positive number, said 0.0001, and obtain the approximate solution of DEMATEL.

The contribution of this paper is twofold. First, it provides a general formula for calculating the result of DEMATEL. Compare with the revised approach provided by Lee *et al.* (2013), the proposed method provides a closed-form expression and flexible way to deal with the interaction effect in DEMATEL. In addition, we incorporate the concept of the interaction diminishing effect into DEMATEL to calculate the total relation matrix. This property gives DEMATEL more flexibility to consider more realistic problems.

Second, this paper proposes extra indicators, including *O-I index*, *closeness centrality*, *betweenness centrality*, *eigenvector centrality*, to help decision-makers to understand the insight of the network. The traditional DEMATEL only uses unscaled *in-degree* and *out-degree centralities* to understand the relative influence/importance of criteria. However, other factors should be considered to fully judge the relative influence/importance of criteria. For example, from the perspective of closeness centrality, nodes having a high closeness centrality are nearby all other nodes and have advantages in accessing resources in a network or having a good overview of the agents in a network.

In order to compare with the conventional DEMATEL technique, we derive the indicators of DEMATEL as shown in Table 2.

Table 2. Full direct/indirect influence matrix in the conventional DEMATEL

<i>F</i>	<i>Column Sum</i> (<i>In-degree</i>)	<i>Row Sum</i> (<i>Out-degree</i>)	<i>Prominence</i> (<i>Row Sum + Column Sum</i>)	<i>Relation</i> (<i>Row Sum - Column Sum</i>)
S_1	2.038	1.561	3.599	-0.477
S_2	2.850	1.281	4.131	-1.569
S_3	0.779	2.107	2.886	1.328
S_4	2.992*	1.916	4.908*	-1.076
S_5	0.906	2.548*	3.454	1.642*
S_6	2.133	2.286	4.419	0.153

According to Table 2, it can be seen that the conventional DEMATEL indicates S_4 is the most important factor (based on Prominence) and S_5 is the most important net causer (based on Relation). However, if we compare the corresponding indicators in Table 1, we can find S_6 is the most important factor and S_3 is the most important net causer. Hence, we can conclude that the result of DEMATEL highly depends on the value of alpha and, therefore, enhances the importance of the proposed method.

In this paper, we propose a general DEMATEL with a closed-form solution to deal with the convergence problem in DEMATEL. The proposed method incorporates the concept of the interaction diminishing effect distinguishes the proposed method from others. It makes DEMATEL more flexible to consider realistic problems. In addition, instead of simply using *out-degree* and *in-degree centralities* in the traditional DEMATEL, we propose five extra indicators for decision-makers to understand the insight of the network. In addition, the conventional DEMATEL is a special case of the proposed method while alpha is equal to 0. Hence, the proposed method can be considered as the generalization of DEMATEL.

Although we extend the DEMATEL technique to consider more general situation and develop extra indicators to understand the insight of the network structure, several limitations of the proposed method are described as following. First, the proposed method ask an expert to quantify the strength of each relation. However, the realistic situation is usually subjective uncertainty. Hence, the extension of the proposed method under fuzzy environments should be considered. Furthermore, the proposed method added a new parameter, alpha, to reflect the interaction diminishing effect. However, the calibration of alpha should be carefully studied, because it may affect the final result significantly. These limitations can be studied further to consider more complex situations.

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