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MATHEMATICAL MODELLING OF MODIFIED CELL DELINEATION STRATEGY IN PACKET SWITCHED NETWORKS

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Abstract. The paper proposes a new mathematical model of Cell Delineation (CD) strategy in any Packet Switching technology when Data Units (DUs) are of constant length. A special strategy that differentiates between synchronization failures and other channel errors with the sufficiently high confidence level has been proposed, analyzed and optimized. The mathematical analysis of the strategy is presented on the Asynchronous Transfer Mode (ATM) network example. The CD cycle implementation time is discussed and analyzed. The numerical results are presented for the case of the standard CD protocol.

Key words: Mathematical modelling, computer networks, packet switching, synchronization

1. The Basics of Packet Network Synchronization Model

Packet synchronization is one of the critical issues in Packet Switching Networks. Synchronization is the mechanism via which the correct boundaries of the packet are properly recognized and identified. The two main conceptual Packet Switching Networks that are commonly used today are: 1) the fixed cell networks, such as the ATM networks; 2) the variable packet length networks, such as the Internet, Protocol (IP) based networks, the General Packet Radio Service (GPRS) or Frame Relay (FR) networks.

In the fixed packet networks, the size of the cells is constant as well as their internal structure, whereas in the variable size cells networks, the cells are separated from each other by a well defined indicator flag located at the leading edge of the packet and/or after its trailing edge.

The present paper will concentrate on the fixed packet network family, while the forthcoming study will extend the current results to the variable size packets networks. The synchronization is so important that when the network fails to properly recognize the cell or packet boundaries, it is out of synchronization and the network is uncontrollable and its contents is missed. More specifically, in the variable size networks the packet boundaries are defined by one of the following mechanisms:

1. A special indicator flag located in the packet beginning and a special field that indicates packet length. This mechanism is used in IP networks;
2. The start and the end flags patterns determine the correct packet boundaries, such as in FR networks.

In the constant packets technologies (for example, ATM networks) the packet length is constant and a packet is called a cell. In the fixed packet networks a cell boundaries are maintained by using the clock synchronization between sender and receiver parties. Thus, the cell length is constant and it is determined by the number of time slots that correspond to the transmission of constant length cell. Therefore, one of the main reasons of the synchronization failures is the discrepancy between the sender and the receiver time clocks. Incorrect determination of the cell boundaries causes distortion not only of a single cell, but overdue stream of cells following it. Trivially this effect causes the cell loss as well as information loss. The synchronization failure existence is recognized by the receipt of a long sequence of the erroneous cells. In general erroneous cells can be caused not only from synchronization failures but also from other reasons such as noisy environment. The basic difference between the synchronization failures and other channel errors can be usually cleared according to the following principle: the receipt of numerous sequential upset may be classified in high probability to be caused by cells synchronization problem occurrence, while the random bit errors, bit loss or bit insertion, usually has a local effect and destroys the current cell without affecting the long cells' sequence. Due to the fact that any synchronization failure destroys several sequential cells, the synchronization failure treatment should be of the homogeneous type applied on the sequence of the erroneous cells.

The loss-of-synchronization can be recovered by two basic methods:

1. The first one is based on the search of the correct cell boundaries by using the bit shifts operations. This mechanism will be discussed later.
2. The second one is the traditional Automatic Repeat Request (ARQ) retransmission techniques applied to a number of the non-sequential cells. It should be noted that the pay-off of the synchronization recovery using the bits shifts in one or more of cells loss, while the retransmission of numerous cells leads to the significant deterioration of the channel utilization rate and additional waste of the network resources such as bandwidth, memory, etc.

The current paper offers a near optimal approach for identifying the synchronization problem cases out of the other failures. Using this procedure, the paper offers a general process for synchronization loss recovery for constant packet size networks. This process can be optimized for specific network configurations. It should be noted that these results will be further extended to the general variable size packet networks in the forthcoming papers.

We will start our analysis by a detailed definition of the synchronization problem in fixed packet networks and will set-up the mathematical grounds for this problem identification and the preliminaries and tools needed for its solution. This will be followed by a detailed analysis of various sequential erroneous cells situations. This approach offers a mechanism for differentiation between non-synchronous and other erroneous cells with a sufficiently high confidence level.

2. The General Synchronization Problem Set-Up

As was mentioned before, we shall be dealing now only with synchronization issues in fixed size packet networks. A success of this approach is a must before extending it to general Packet Switching Networks.

2.1. Mathematical set-up of the synchronization problem

The cell size in the fixed packet network is denoted by $L = m + n$ bytes, where m bytes is the size of the packet header, and n bytes is the size of the payload or the contents which trails the header (for the ATM networks $L = 53, m = 5 \textit{ and } n = 48$) [8].

Among other parameters (such as routing information, QoS and management information) the header contains an indicator for the correctness of the header contents. This indicator, for most of the network technologies, is of the CRC type. It is usually located in the last *m-th* byte of the header. This one-byte-size CRC indicator is usually of the Header Error Control (HEC) type, where it is constructed using the Cyclic Redundant Check $CRC\left(2^{m-2}\right)$ of the leading $(m-1)$ bytes of the header (for example, the HEC in the ATM is $CRC(8)$) [1]. The network system verifies that the computed CRC value of a given cell's header is equal to the HEC carried by this header itself. Once this equality fails, there is a reasonable suspicious for an error in the given cell: the error source can be of type 1 or type 2. The failures of type 1 indicate the network synchronization failure, while the errors of type 2 correspond to the different channel errors such as bit or bits errors, loss or insertion. The type 2 errors influence cell only locally, and are not in the scope of this paper. In case where the consecutive cells' HECs are in error, the suspicious for the type 1 error increases as it shown in Figure 1. The reason of this phenomena is that the synchronization failure influences the large number of the sequential cells, while the random errors of type 2 influence locally a given cell. The longer the sequence of erroneous cells is, the larger is the confidence that it is a type 1 error. Practically, it is convenient to define a threshold number beyond which the number of erroneous cells indicate the synchronization problem.

This case is the main issue of this paper. Mathematically, and traditionally, it is convenient to represent the synchronization problem using the Finite State Machine (FSM) facilities. The FSM gets the input stream of cells, and

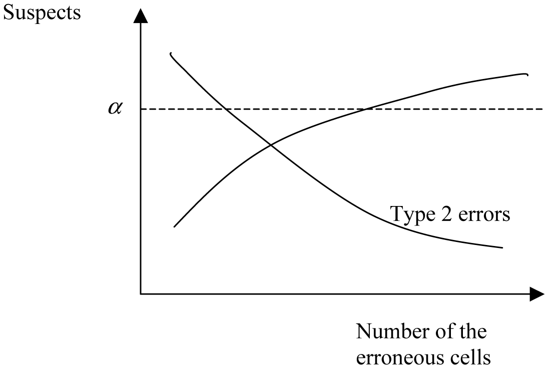


Figure 1. Suspects for different error types as a function of the erroneous cells number.

maintains a buffer b which may contain several consecutive wrong cells. The most common case is when the network is well synchronized, and this state is called the $SYNCH$ state. The $SYNCH$ state processes are defined as follows:

```
 1. k =0.
 2. Start a new cell.
 3. Set $m-1$ bytes from the input stream.
 4. Calculate CRC.
 5. Compare to HEC.
 6. if ( HEC correct) go to 2 for the next cell
    else
 7.    Move incorrect cell to accumulated bucket of wrong cells $b$.
 8.    Number of wrong cells $k++$.
    end if
 9. if ( $k \leq \alpha$) go to 2
    else
10.    Go to $HUNT$ state.
    end if
```

The *HUNT*-state is activated once the size of b exceeded the value of α. Being in this state the network system starts to resolve the wrong cells issue as if they were caused by a synchronization problem. When the network is in the state of being not-synchronized (*HUNT*-state), the circuit searches for the HEC byte by calculating the $(m-1)$ previously received bytes using $CRC\left(2^{m-2}\right)$ and compares it against the current byte. If a valid HEC byte is found, using the first HEC byte location as a reference point, and if δ consecutive HECs are found ($\delta = 7$ in the ATM standard), the synchronization is achieved. Once synchronization is achieved through HEC byte search, HEC errors are detected. These errors, their handling and control are the subject of this paper.

The following assumptions are made in the presented model: 1) each cell is transmitted in a different time slot; 2) the cells are transmitted continuously head-to-tail; 3) the time slot occupancy is not checked at the routing related processes low layer; 4) the synchronization problem which is related to the clock shift is constant at some reasonable time interval.

3. The Problem Statement

The topic of this paper is to offer a strategy, algorithms and relevant techniques for properly recognize and maintain cell or packet's boundaries. In case that the network fails to properly recognize the cell or packet boundaries, it is said that the packet network is out of synchronism. As a matter of fact, the suggested strategy is designed to tackle and handle general error packets, where one of our major objectives is to differentiate in real time between synchronization failures and other noisy environment channel errors. The meaning of real time here is the heart-bit of the network itself, meaning that any cell or packet correction will not upset the overall delivery rate of the packets. This means that the implemented synchronization failures recovery and correction must be fast and homogeneous. A new advanced strategy for cell synchronization recovery and other noisy channel errors correction is proposed, analyzed and optimized. Several tests and examples conducted in an ATM network environment fully support this new approach, its derived technologies and implementation.

3.1. Cell delineation process description

The CD process is the mechanism via which the cell boundaries are properly identified. The standards for ATM networks offer the most advanced mechanism for the CD process. Here the standard CD protocol is defined in ITU specification [1] using the Finite States Machine (FSM). According to this approach, any ATM cell that is out of synchronism enters a specific state of the FSM, and then, in time, after applying the relevant algorithm its state changes, till it gets into the proper state for being releasing back to the network. The parameter of the ATM cell that indicates out-of-synchronism situation of the cell is the Header Error Control (HEC), and it is embedded in the ATM cell header.

Following this protocol's description, the three discrete states are considered in the current paper (Figure 2):

1. The *SYNCH* state. This state corresponds to synchronous operation in cell reception process at the entity-destination of any data link. This state is defined by receipt of sequential cells with correct HECs.
2. The *HUNT* state. This state is reached if $n \geq \alpha$ consecutive non-correct HECs have been detected. In this case, all these n cells are entering into the *HUNT* state. Now, the n cells in this state are properly recovered by applying a sequence of bit shifts until the cells with correct HECs are discovered. Once this algorithm is positively terminated, all the corrected cells change their state into the *PRESYNCH* state. By definition of this

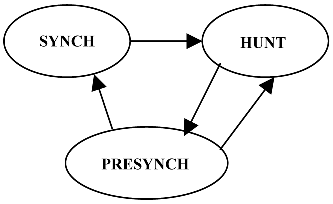


Figure 2. CD process structure.

process, some cells loss is possible at the *SYNCH* → *HUNT* interface. The parameter α determines transition delay and will be discussed later for our strategy.

3. The *PRESYNCH* state. The cells get into that state after they have visited the *HUNT* state and the algorithm in this state has positively discovered these out-of-synchronism cells. Once at least δ consecutive cells with corrected HECs were accumulated in this state they are automatically mapped into the *SYNCH* state. This process of the cells transition into the *SYNCH* state imposes some delay on the overall correction process. This delay is necessary because the sequence of correct HECs could be caused not only by correct cells synchronization, but also by undetected errors in the cell header decoding process.

This standard CD model is not suitable in the case of the non-reliable noisy channel. For the ATM networks, according to the standard, any multiply bit errors detected in the cell header will cause cell to be dropped. However, it is quite dangerous, because the cell boundaries are not clear and any drop may cause more additional problems. The origin of this problem is the fact that synchronization failures and other channel errors are indistinguishably treated in the same way leading to too much cell drop and information transfer lose. It is critical that these two different types of errors will be treated differently resolving the confusion of these two errors, with proper and relevant error recovery and correction, at least in a high probability execution. For this more, the α and δ parameters should be a function of the specific network and its history. That means that the couple (α, δ) should not be static (as in the ATM standard) and should be allocated in a dynamic fashion. Therefore, the limited CD protocol defined by the set of three discrete states (as in ATM networks) is appropriate in the following above cases:

- High quality fiber optic channel with extremely small Bit Error Rate (*BER*).
- SONET/SDH technology with internal cells protection [8].

Hereafter we present a more precise and adequate CD strategy that contains all the four possible discrete states. An additional *ERROR* state is added to the previously described CD protocol in order to distinguish, and then trying to recover and correct random errors in the constant ATM cell Header. This modified CD process is presented in Figure 3.

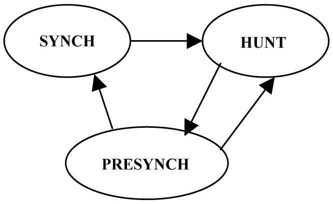


Figure 3. The modified CD process structure.

The proper model for the various modified CD transitions, namely *SYNCH*→ *ERROR*, *HUNT*→*ERROR*, *ERROR*→*HUNT*, *ERROR* → *PRESYNCH* will be discussed later. Nevertheless, the newly added *ERROR* state enables the system to distinguish between two incompatible processes, the synchronization failures and other channel errors, laying the framework to distinguish between the errors generated by the different sources. In order to have this model to basic mathematical closure requirements, there is a need to add the fifth discrete state, the *FAILURE* state which is relevant to the channel degradation problems and errors. In contrast to the other states, this state is basically an absorptive one (Figure 4).

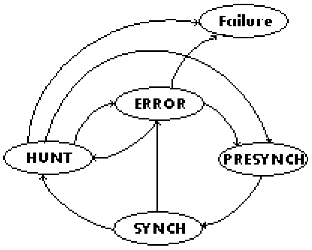


Figure 4. The broadened CD process structure.

The immediate realization of the channel recovery procedure by means of software and hardware tools is necessary in this case. Such situation happens when the network is very sensitive. By virtue, the network health problem should be solved immediately in real time. For example, the multimedia applications require such distinct approach.

This paper studies and optimizes the four-discrete states CD protocol. We guess that such protocol execution is appropriate to numerous cases.

3.2. Markov chains mathematical set-up

Markov chain approach is used in order to describe behaviour of the model that assumes dependence between sequential events. As a matter of fact, there are numerous applications of Markov chains and semi-Markovian processes in the analysis and optimization of Data Communication protocols and in the traffic modelling.

The CD process is essentially, semi-Markovian. In our opinion, the most reasonable mathematical tool to specify the arrival process of the correlated cells is the ergodic simple Markov chain. The *HUNT*, *PRESYNCH* and *SYNCH* CD process states determine the Markov chain discrete states. Transmission time from one discrete state to another is random and is described by a set of distribution functions. We have found the following reasons to apply this approach in this paper:

1. By introducing a semi-Markovian model it is possible to analyze with facilities the CD recovery time in the real time. Therefore, the CD process characteristics can be determined not only as the static ones, but also as the dynamic characteristics that refer to the specific time intervals.
2. The protocol semi-Markovian model allows us to take into account statistical correlation between the sequential discrete states of the process. In real communication systems, any protocol operations to be performed currently depend on the system's previous state.
3. The presented model allows us to differentiate the different sources of the CD failures. The following failures are meant: distortion of the cell Header that is caused by channel errors, synchronization failures and channel failures.

However, in many applications up to now considerable difficulties arise if it is necessary to derive not only average values, but also the exact values of the protocol characteristics distribution moments. Here any protocol characteristics is treated as a random variable with a distribution function (DF). We developed a probabilistic model and an appropriate computer procedure that allow us to derive distribution moments of any order in application to the additive protocol characteristics and consequently to construct the corresponding DF for the standard three-states CD protocol.

We propose a special methodology for exact determination of the following random variables distribution functions: 1) synchronization establishment time T_c, 2) time occurrence in the non-synchronous state *HUNT* T_{ns}. Based on the mathematical expectations of T_c and T_{ns} the final probabilities of the system being in synchronous and non-synchronous state correspondingly are determined. These probabilities are the functions of the arguments α and δ. The algorithm for the calculation of any order distribution moments and the determining of the above-mentioned random variables is developed. The appropriate analysis of the Modified four-states CD protocol will be carried out in the forthcoming research.

4. The Modified CD Protocol Features

The following features are required considering the modified CD strategy:

1. CD protocol execution has to be fast and effective. The probability of non-correct decision about synchronization failure and synchronization establishment must be very small considering a certain confidence level.

2. A single stable state *SYNCH* exists. Besides, the other states are non-stable and their time occurrence is extremely smaller than the *SYNCH* state time occurrence.

3. The α parameter should be assigned dynamically considering the following arguments: a) error flow model of the noisy channel in the respect to the current *BER*; b) channel errors history; c) traffic type (Constant Bit Rate (CBR), Variable Bit Rate (VBR), Available Bit Rate (ABR), or Unspecified Bit Rate (UBR)).

4. The cells are divided in the two types: a) Black (B-type) erroneous cells. Considering ATM standard cells, the B-type cell is, in essence, any cell with detectable error in the cell Header. It should be mentioned that the error nature may be a cell synchronization failure as well as any error caused by a noisy channel; b) White (W-type) cells that are correct ones.

5. The random number of the sequential B-type cells is studied within a certain time window T. Usually, the T near-optimal choice is urgent in order to create the effective and robust delineation strategy.

6. Denote by B' and W' the average number of B and W-type cells received in the T range, respectively. It is assumed that within a given T period

$$B' << W'. \tag{4.1}$$

Otherwise, the presented strategy failures.

7. The strategy that is enabled to distinguish between non-synchronous and other erroneous cells among α sequential B-type cells, should be defined in the precise way.

The problems stated and solved in this paper is:

1. To determine the optimal minimal value $b_{\min}$ of the B-type sequential cells that identifies with the certain confidence level the synchronization failure occurrence.
2. To implement the synchronization recovery strategy in real time in the near-optimal way considering the requirements to the following parameters: a) error flow model of the noisy channel in respect to the current *BER*; b) synchronization failures model considering the current value of the cell non-synchronous state final probability as well as the synchronization failures history. Note that the synchronization failures have memory nature. Hence, they can be described by the Markov model; c) different traffic types and, besides, users demands on QoS characteristics.

5. Modified CD Strategy

Let us determine the CD decision scheme in the following manner. Assume that within a given T range $\alpha \geq 0$ consecutive B-type cells have been detected in the receiver. It should be reminded that the $b_{\min}$ purpose is to prevent confusion in the receiver decision about synchronization failures that are not related to the other channel errors. For larger values of $b_{\min}$ the probability of the correct decision about synchronization failure is also larger. The modified CD strategy has the following policy for $b_{\min}$:

1. The value of $b_{\min}$ with respect to its variance σ :
   a) If the number of the consecutive B-type cells $\alpha < b_{\min} - k\sigma,\ k \in N$, then the received cells arrive immediately to the *ERROR* state. It is assumed that the cells non-correctness is caused by different error sources that are not related to the synchronization failures. Here $\sigma > 0$ is the $b_{\min}$ variance and k is a certain natural number.
   b) If $b_{\min} - k\sigma < \alpha < b_{\min} + k\sigma$ then the damaged cells pass to the *HUNT* and *ERROR* states simultaneously. The CD and error recovery procedures are applied simultaneously by definition. The reason is that the error source cannot be determined with the sufficiently high confidence level. Thus, the additional operations executed at the receiver provide cell delineation as well as random errors recovery and correction. Therefore, cell loss is prevented in the *SYNCH* → *HUNT* and *SYNCH* → *ERROR* interfaces. In order to keep the original cells order, the recovered cells transmitted to the *PRESYNCH* state, should be assigned with the sequential numbers. We guess that in the case of ATM cell the Generic Format Identifier (GFI) field (see [8]) or the reserved combination in the Payload Type field may contain the recovered cells numbers.
   c) Finally, if $\alpha > b_{\min} + k\sigma$, then the decision about synchronization failure is more stable and, therefore, the system passes to the *HUNT* state.
2. For the case were $\alpha > b_{\min} + k\sigma$ B-type cells pass from the *SYNCH* state to the *HUNT* state, any number of B-type cells have been received within next time windows T pass to the *HUNT* state as well. The reason for that is that as the error source probability is larger, the synchronization failures that should be recovered by means of per-bit shifts are greater. As a matter of fact, the traffic modelling will be very useful in the error bursts analysis and prediction.
3. In the special case of the error sensitive network the transition *HUNT* → *FAILURE* and *FAILURE* → *HUNT* should be provided immediately if the following conditions are fulfilled simultaneously:
   a) After maximal acceptable number of bit shifts L' the correct cell boundaries have not been determined in the *HUNT* state;
   b) The error recovery procedure is not capable to recover random errors or error bursts. The maximal value of the acceptable shifted bits is $L' = 212$ by definition. Besides, it is not worth to execute the shifts number that exceeds the half of the ATM cell length. Note that the standard cell length in the ATM network is 53 bytes, or 424 bits.
4. The transitions *HUNT* → *ERROR* and *ERROR* → *HUNT* are possible if there are no strict limitations on the CD process time execution:

a) The transition *ERROR* → *HUNT* is relevant if the control relations are not able to recover errors. Such situation is typical for any value α of the B-type cells;
b) The reverse transition *HUNT* → *ERROR* takes place in the opposite case when the *HUNT* state operations have been failed in the determining of the cell/cells correct boundaries.

In the next section the advanced operations in the *HUNT* and *ERROR* states will be introduced.

5.1. Dynamic assignment of $b_{\min}$ parameter

Let us pass now to the $b_{\min}$ parameter determination, which is, in essence, the basis in the problem of the CD strategy near-optimal implementation. Moreover, the correct and precise determination of $b_{\min}$ variance range $k\sigma$ is of the special interest. The reason is that it influences significantly the CD strategy parameters such as:

1. Probability of non-correct decision about synchronization failure or establishment respectively.
2. Robust synchronization recovery after the synchronization failure takes place.
3. Minimization of cell loss probability, etc.

We aim towards the minimization of the CD cycle average time `average(Tc)` as well as variance and distribution moments of any higher order. On the one hand, if `average(Tc)` is small, then the transition is erroneous with a rather high probability. On the other hand, in the reverse case, the cell synchronization establishment time is too large. The issue here is to present the optimal trade-off of the discussed above parameters. The following assumptions are made:

1. Denote by P_i the probability for i consecutive erroneous cells. Determine the average number of the erroneous cells as

$$b'_{\min} = \sum_{i=1}^{\infty} iP_i. \tag{5.1}$$

Then the minimal number of the sequential non-correct cells that is sufficient to indicate that the synchronization failure happens, is equal to

$$b_{\min} = b'_{\min} + k\sigma. \tag{5.2}$$

Here k is a specific tolerance coefficient, and its values will be discussed later.

2. The natural way in computing $b_{\min}$ and σ is based on the Cell Error Rate (CER) range values. Clearly, CER depends on BER in the noisy channel. Then the discussed above minimal number of the sequential B-type cells is the function of the average number of the average $CERs$ within a specific time window T

$$b'_{\min} = \sum_{i=1}^{n} b_{\min_i} CER_i. \tag{5.3}$$

3. The more precise approach of the synchronization failure identification in the sequential B-type cells $b'_{\min}$ is based on the applying of two kinds of statistics [2]: a) short term statistics that describes the current noisy channel status and the corresponding $b_{\min}$ value; b) long term statistics that is related to the channel history.

The Markov chain approach will be useful in prediction of the $b'_{\min}$ value. Assume, that the channel history is described by the following two parameters:

1. Probability transition matrix $M = (m_{ij})$ of the simple regular Markov chain. Its elements are, essentially, the conditional probabilities of the CER value CER_i on condition that at previous step $CER = CER_j$. The Markov chain step is defined as cell transmission time that is assumed to be constant. The reason is that ATM cell is of constant length. The matrix M elements should be determined on real statistical data of the communication channel.
2. Markov chain initial probabilistic vector $\pi(0)$ that determines the initial CER distribution. Then the CER distribution at any step N is computed as $\pi(N) = \pi(0) M^N$. Moreover, based on the generating functions approach, the final distribution of the different CER values should be received in the precise analytical form [7]. The inherent convergence condition is $\pi(N) \to \pi(\infty)$, $N \to \infty$. Thus, the matrix M must satisfy the following condition: M^N is monotonous function of the argument N.

The current $b_{\min}$ value is computed in the following way:

$$b_{\min} = w\, b'_{\min} + (1 - w)\, \varPhi(N). \tag{5.4}$$

Here $0 < w < 1$ determines the relative weight of the short term and long term statistics respectively. $\varPhi(N)$ is a linear operator that transforms $\pi(N)$ into a scalar. $\varPhi(N)$ can be defined in different ways, for example:

$$\varPhi(N) = \frac{1}{N} \sum_{i=1}^{d} \pi_i(N), \quad \varPhi(N) = \max(\pi_1(N), ..., \pi_d(N)),$$

where $d = dim(M)$.

5.2. Statement and discussion of the $b_{\min}$ optimization problem

Our goal is to define the minimal cells number required to decide about synchronization failure considering the limitations applied on: 1) synchronization failure final probability γ, 2) probability p_{nc} of non-correct decision about synchronization failure, 3) average CD cycle time $average(T_c)$, 4) CD cycle time variance V.

The corresponding objective function is defined as:

$$F(b_{\min}, k) = \left(b'_{\min} + k\sigma\right), \quad k \in N, \ \sigma > 0. \tag{5.5}$$

Here σ is the $b_{\min}^{'}$ variance that is computed in the previous section and, actually, it has a constant value.

The limitations imposed on the objective function $F(b_{\min}, k)$ are:

$$\gamma \leq \gamma', \quad T_c \leq T', \quad V \leq V'. \tag{5.6}$$

As a matter of fact, the k parameter depends on the synchronization failures model and, besides, on the synchronization failure final probability γ. On the one hand, the more is γ, the higher is probability that the discussed number of the B-type cells are non-synchronous. Therefore, the $b_{\min}$ confidence level defined by $k\sigma$ is smaller. On the other hand, the rarer the synchronization failures occur, the confidence level of $b_{\min}$ is larger. In order to satisfy the limitation (5.6) the functional dependence of k on γ should be defined:

1. $k = g(\gamma)$ is a decreasing function such that

$$\lim_{\gamma \to 0} (g(\gamma)) = 0, \quad g'(\gamma) < 0.$$

2. The exponential form $k = A\exp(-B\gamma) + C$ is suitable in case of the network self-similarity. We mean that the network behaviour is the same during any short time interval as well as during any long time interval. Therefore, equation (5.5) is presented as follows

$$F(b_{\min}, k) = b_{\min}^{'} + \sigma(A\exp(-B\gamma) + C).$$

3. The A, B, C are the non-negative constants, whereas $b_{\min}^{'}$ and σ should be determined based on the real statistical data of the communication channel.
4. The conditions (5.2)–(5.3) are mainly influenced by the traffic type.
5. The solution of optimization problem will be based on the dynamic programming approach that is based on the Lagrange multipliers.

6. Example of the Three-States CD Strategy Realization

In this section we present the example of the standard CD protocol realization in ATM networks. We present a general analysis of the final probability occurrence in the *HUNT* state p_{ns}, T_c and of T_{ns} considering the following parameters:

1. Different error flow models (Markov and Bernoulli) of the communication channel [4].

2. Basic and advanced *PRESYNCH* protocol [3]. The existing basic CD protocol is characterized by possibility to perform non-correct transition *PRESYNCH* → *HUNT*. This operation is executed each time when any cell with incorrect HEC in the *PRESYNCH* state has been detected. This non-correctness is caused by the situation when the synchronization system is robust, but a random detectable error happened in a cell header. To improve the basic CD protocol a special decision mechanism has been proposed.

This mechanism evaluates a conditional probability of the correct transition *PRESYNCH* → *HUNT* on condition that a cell with incorrect HEC has been received. This mechanism takes into account the a priori probabilities of the events that result in receipt of cell with incorrect HEC.

3. Different models of the synchronization failures: self-recovery and non-self recovery synchronization failures [4]. Self-recovery synchronization failures are recovered on the bit synchronization level and described by the Bernoulli model without memory dependence. Therefore, synchronization failure in any cell does not cause synchronization failures in the cells sequence. In case of the non-self-recovery synchronization failures, synchronization failure in any cell leads immediately to synchronization failures in the following cells. These failures are described by the memory Markov model. Thus, these failures can be recovered only in the *HUNT* state.

6.1. Formal model

The GERT method was introduced by Pritsker [11]. It was used for solutions of economic and planning problems in [10]. The GERT method is applied on a stochastic network base. According to [10], a network (graph) $G = \{V, A\}$ which consists of vertices set V and arcs set A, is defined as stochastic if the execution of a specific set of arcs actions is sufficient for a whole network project execution. In stochastic networks, executions of actions related to a whole set of entrance arcs of any vertex, is not a necessary condition for this vertex's execution. As follows from this condition, the operations with feedback can be carried out in this model. Thus, the stochastic network model assumes the occurrence of the cycles or of the loops. Additionally, execution time of any vertex is considered to be random and is described by a probabilistic distribution.

Considering the problems that will be discussed below, the following initial conditions are assumed:

1. The entrance function of each vertex is characterized by the feature that the vertex's operation executes if any of its entrances receives a signal about the action only on one of the possible arcs.
2. Vertices exit functions are classified as deterministic and probabilistic. In the first case, all the exit arcs actions of the vertex are executed, if corresponding vertex event has taken place. In the second case the equation $\sum_{k=1}^{z} w_k = 1$ holds if the vertex event occurs. Here z is the number of the given vertex exit arcs and w_k is the probability of the k-*th* arc's action execution.
3. Processes related to the operations in separate vertices are independent. Thus, the method proposed does not deal with the communication network protocols that are described by complex Markov chains.
4. Protocol characteristics are additive.

The conditions (5.1)–(5.4) allow us to treat the stochastic networks as *GERT*-networks and vertices as *GERT*-vertices as they were defined in [11]. Let us describe the CD protocol by an appropriate stochastic network $G(V, A)$.

The network consists of the *GERT*-vertices set and of the *GERT*-arcs set. For each vertices pair we define a set of conditional probabilities (in case of discrete distribution) or probability densities (for continuous distribution case) $f(g_{xy})$ of the random variable g_{xy}, that is related to the studied protocol characteristic. The first case is typical for analysis of reliability characteristics such as a) number of false synchronized cells; b) number of lost cells during synchronization establishment stage; c) number of synchronization failures during a specific time interval.

The second case is appropriate to analysis of the time characteristics such as synchronization establishment time T_c, and time occurrence in the non-synchronous state *HUNT* T_{ns} during a synchronization establishment cycle. As a matter of fact, such differentiation is rather conditional because the limit transition from the discrete random variable to the continuous one is permitted if the number of experiments (or the number of Markov chain steps) is sufficiently large [2].

Let us define now a set of conditional Moments Generating Functions (MGF) of a random variable g_{xy} with argument s [9]. In case of continuous random variable g_{xy}

$$M_{xy} = \int_{G_a}^{G^b} \exp(sg_{xy}) f(g_{xy})\, dg_{xy},$$

where $[G_a, G^b]$ is g_{xy} variation range. In case of discrete g_{xy} distribution

$$M_{xy} = \sum_{R_{xy}} \exp(sg_{xy}) f(g_{xy}).$$

Here R_{xy} is a set of g_{xy} realizations under the movement across the arc between the vertices x and y.

MGF is the basic tool for any order initial distribution moment derivation of the corresponding random variable. The *i-th* initial distribution moment of a given random variable is determined as the *i-th* order derivative of MGF in the point $s = 0$ [9]. In [10] the set of W–functions is defined as

$$W_{xy}(s) = p_{xy} M_{xy}(s), \quad x, y \in V.$$

W_{xy} contains the complete information about transition between the vertices x and y in the stochastic network. Hence, the set of W_{xy}-functions $\{W_{xy}\}$ is, essentially, the basis for determining the general W-function of the stochastic network G. Define $W_0(s)$ as W-function that is appropriate to the general CD protocol execution. Thus, the *GERT* network is considered as a combination of basic element structures. If W-functions of all these typical structures are known, the W-function corresponding to all branches of the *GERT* network can be constructed. Based on Mason's rule [10], any *j-th* initial distribution moment μ_j of any protocol characteristic treated as a random variable is determined from MGF of network by producing the *j-th* derivative:

$$\mu_j = \left.\frac{d^j M_0(s)}{d^j s}\right|_{s=0} = \frac{1}{W_0(0)} \frac{d}{d^j s} W_0(s)\left.\right|_{s=0}.$$

6.2. W-functions of the typical topological structures

Let us define W_0 as general W-function that corresponds to the CD process execution. Let us assume that W_{12}, W_{23}, W_{32} and W_{31} are the W-functions that correspond to the transitions *SYNCH* → *HUNT*, *HUNT* → *PRESYNCH*, *PRESYNCH* → *HUNT* and *PRESYNCH* → *SYNCH* respectively.

Considering the probabilistic graph corresponding to the CD process deployment the following typical fragments of the stochastic network that satisfy the conditions of possession to the *GERT*-networks are distinguished.

6.2.1. Consecutive connection of arcs

Let us assume that there is consecutive concatenation of $n-1$ arcs divided by n *GERT* vertices. This structure depicts the transition in the CD cycle in the simplest case *SYNCH* → *HUNT* → *PRESYNCH* → *SYNCH*. Since the protocol characteristic is additive and MGF of the sum of independent random variables equals to their MGFs multiplication, i.e., $M_{xy} = M_{xi}\, M_{iy}$, the following equation is correct:

$$W_{xy} = p_{xi}\, M_{xi}\, p_{iy}\, M_{iy}. \tag{6.1}$$

This result may be generalized for an arbitrary number of arcs, i.e. the resulting W-function equals to

$$W_{1n} = \Pi_{x=2}^{n-1} W_{1x}\, W_{x,x+1}$$

Further we will write W instead of $W(s)$.

6.2.2. Loops or cycles that prevent the next arc execution

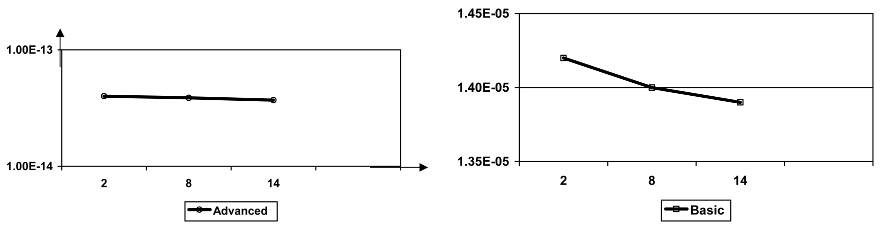
This structure corresponds to the multiple transitions *HUNT* ↔ *PRESYNCH* within a given CD cycle. The W-function that corresponds to the different number of the transitions *PRESYNCH* → *HUNT* is determined as

$$W = \sum_{i=1}^{\infty} \left(W_{32} W_{23}\right)^i .$$

Clearly, if the condition $\mathcal{O}(B') \ll \mathcal{O}(W')$ is not hold, then the corresponding W-function could not be structured because the infinity of the loop $HUNT \leftrightarrow PRESYNCH$ causes the infinite CD cycle time.

Based on the general topological equation (Mason's method) [10], the resulting W-function corresponding to the CD process stochastic network execution is defined as

$$W_0 = W_{12} W_{23} W_{31} + W_{12} \sum_{i=1}^{\infty} \left(W_{23}^2\right)^i W_{31}.$$



a) b)

Figure 5. p_{ns} as function of δ: a) case corresponds to the advanced *PRESYNCH* protocol, b) case corresponds to the basic *PRESYNCH* protocol, here $\alpha = 8$, Markov error flow model, $BER = 10^{-6}$, Poisson model of synchronization failures, $\lambda = 10^{-12}$.

6.3. Numerical results and discussions

The presented results show the following features of the general CD process:

1. The final probability of the cell non-synchronous state p_{ns} is the decreasing function of the α and δ arguments (see Figure 5).

2. Considering the basic and the advanced CD protocols, it is evident that any order distribution moments are lower in the advanced protocol case. Consequently, p_{ns} is low in this case referring to all spectrum of (α, δ) values (see Figure 5). Moreover, the probability of the wrong decision about the synchronization establishment is significantly low in the advanced protocol case.

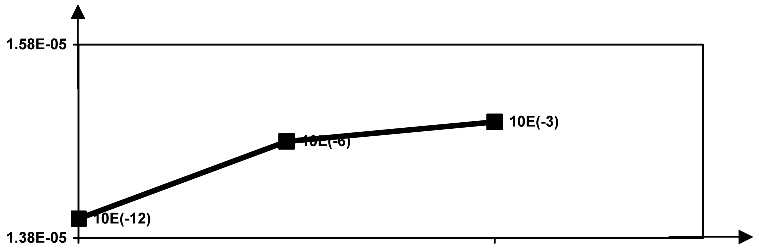


Figure 6. p_{ns} as function of γ, here $\alpha = 8$, $\delta = 7$, $BER = 10^{-6}$. The presented range of γ values is $10^{-12}, 10^{-6}, 10^{-3}$.

3. Referring to both types of error flow models (Bernoulli and Markov) and to both types of synchronization failures (Poisson and Markov) the final probability of cell non-synchronous state p_{ns} increases by the increment of Poisson synchronization failures intensity λ or on synchronization failures final probability γ (see Figure 6).

4. Referring to the self-recovery synchronization failure model and to the non-self-recovery synchronization failure model, the average cycle time

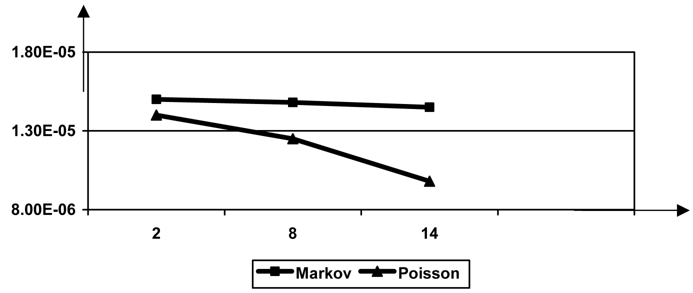


Figure 7. p_{ns} as a function of α, $\delta = 7$. Markov error flow model, $BER = 10^{-6}$, $\lambda = \gamma = 10^{-12}$. Case 1 corresponds to self-recovery (Poisson) model of synchronization failures, case 2 corresponds to non self-recovery (Markov) model of synchronization failures.

$average(T_c)$ as well as all distribution moments of any higher order, are significantly higher in the second case. The reason is that the receipt of any non-synchronous cell in the *SYNCH* state automatically causes the receipt of a sequence of the non-synchronous cells. Therefore, the transition time to the *HUNT*state is essentially low in this protocol model. The decreasing function $p_{ns}(\alpha)$ is shown in Figure 7. Evidently, the probability of the non-synchronous cell state p_{ns} is higher in case of non-self recovery synchronization failures model.

The main advantage of the presented *GERT* method is the ability of structuring the general T_c and T_{ns} distribution functions in the precise manner. These functions are based on the whole set of any order initial distribution moments. Therefore, the “distribution trail” can be determined in a distinct way. In this paper we present the normal two parameters T_c distribution functions. Normal distribution is defined by means of the probability density function in the following way [9]:

$$f(t) = \frac{1}{2\pi\sigma} \exp\left(\frac{-(t-E)^2}{2\sigma^2} \right),$$

where E and σ are mathematical expectation and variance of t random variable respectively. Normal distribution appropriates to case of the CD process large time duration. In this case the transmitted cells number is very large. We apply the generalization of the central limit theorem [2].

Theorem 1. *If a recurrent event Ψ takes place and its mathematical expectation and variance are finite, then the number of Ψ appearances in the first k experiments is distributed according to the normal law.*

Clearly, the CD process Markov model satisfies the theorem conditions. Hence, if the transmitted cells number $N \to \infty$ then T_c and T_{ns} distributions follow the normal low. Figure 8 demonstrates T_c normal distribution functions

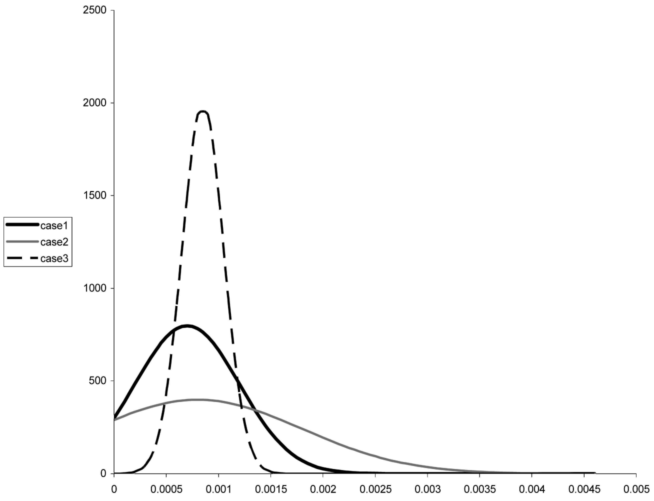


Figure 8. Average CD cycle time *average* (T_c) normal distribution function. Basic *PRESYNCH* protocol. Case 1 corresponds to $\alpha = 2$, $\delta = 1$, case 2 corresponds to $\alpha = 8$, $\delta = 7$, case 3 corresponds to $\alpha = 14$, $\delta = 13$. Markov error flow model, $BER = 10^{-6}$. Self-recovery synchronization failures, $\lambda = 10^{-12}$.

corresponding to a different range of the CD process parameters. In essence, the greater are α and δ values, the T_c mathematical expectation and variance are larger.

7. Appendix B

Calculation Procedure

In order to determine the general W-function W_0 related to the CD process execution let us determine the W-functions W_{12},W_{23}, W_{32}, W_{31} that correspond to the transition from one discrete state to another in the three-state CD protocol. According to the previous definition of the W-function (6.1) the following equality is correct

$$W_{12} = p_{12} M_{12}.$$

Here p_{12} is the unconditional probability of the transition *SYNCH* → *HUNT*. Obviously, $p_{12} = 1$. Consequently,

$$M_{12}(s) = \sum_{i=1}^{\infty} \exp(st_i)\, \pi_{abs}^{SYNCH}(i).$$

Here t_i is the i cells transmission time [4]. Taking into consideration that the *SYNCH* state is described by a simple Markov chain model [4], we conclude that $t_i = it$, where t is the cell transmission time, $\pi_{abs}^{SYNCH}(i)$ is the absorptive component of the probabilistic vector on the *i-th* step that corresponds to the transition *SYNCH* → *HUNT*. The synchronization failure model must be taken into account in order to evaluate $\pi_{abs}^{SYNCH}(i)$. In case of self-recovery synchronization failure $\pi_{abs}^{SYNCH}(i)$ corresponds to the receipt of α consecutive cells with incorrect HECs. Contrary, in the second case, $\pi_{abs}^{SYNCH}(i)$ relates to the receipt of α consecutive non-synchronous cells. Therefore, confusion between the erroneous cells and the non-synchronous cells will be prevented in this case. Essentially,

$$W_{23} = p_{23} \sum_{i=1}^{\infty} \exp\left(st_i'\right) \pi_{abs}^{HUNT}(i).$$

Here t' is the processing time of i bits and it is equal to $t_i' = it'$, where t' is a bit processing time. The absorptive component of the *HUNT* state probabilistic vector $\pi_{abs}^{HUNT}(i)$ contains the following two items. The first item corresponds to the correct recovery of a cell boundary (i.e., the correct cell synchronization) and the second one results from the false synchronization recovery. Obviously, the first case is more likely.

The two possible ways of transitions from the *PRESYNCH* state should be considered in the creation of W_{31} and W_{32}. The first one corresponds to achieving of the *SYNCH* state and making a decision about synchronization establishment [3]. The following equations are presented:

$$W_{32} = p_{32} \sum_{i=1}^{\infty} \exp\left(st_i\right) \pi_{abs_1}^{PRESYNCH}(i),$$

$$W_{31} = p_{31} \sum_{i=1}^{\infty} \exp\left(st_i\right) \pi_{abs_2}^{PRESYNCH}(i).$$

Here p_{32} and p_{31} are the unconditional probabilities of the transitions *PRESYNCH* → *HUNT* and *PRESYNCH* → *SYNCH* accordingly. Clearly,

$$p_{31} = 1 - p_{32} \pi_{abs_1}^{PRESYNCH}(i).$$

Here $\pi_{abs1}^{PRESYNCH}(i)$ and $\pi_{abs_2}^{PRESYNCH}(i)$ are the *i-th* step absorptive components of the *PRESYNCH* state probabilistic vector that correspond to the correct and incorrect completion of the *PRESYNCH* protocol [6]. During a given cycle a different number of the transitions *PRESYNCH* → *HUNT* is possible. The single transition *PRESYNCH* → *HUNT* takes place if a single cell with an incorrect HEC has been detected among δ consecutive cells. The two returns from *PRESYNCH* to *HUNT* are achieved if there are precisely two cells with incorrect HECs among δ consecutive cells. Finally, L transmissions *PRESYNCH* → *HUNT* are sure to occur if $L \le \delta$ incorrect HECs have been recovered among δ consecutive cells.

In case of the basic *PRESYNCH* protocol the following equation is valid:

$$W_{32} = \sum_{j=1}^{\infty} \Big(\sum_{i=1}^{L} (1-G)^i G^{\delta-i} \exp\left(st_j\right) \pi_{abs_2}^{PRESYNCH}(j) \Big),$$

where $G = (P(0.40) + P(1.40))$.

In case of the advanced CD protocol we conclude that

$$W_{32} = \sum_{j=1}^{\infty} \left(1 - \frac{1-G}{U+(1-G)}\right)^{j} (1-U)^{\delta-j} \sum_{i=1}^{\infty} \exp(st_i)\, \pi_{abs_2}^{PRESYNCH}(i). \tag{7.1}$$

Here U is a probability of the synchronization failure during a cell transmission. In case of self-recovery synchronization failures $U = \exp(-\lambda t)$, where λ is the synchronization failures Poisson intensity. In case of non-self recovery synchronization failures $U = \gamma$, where γ is defined as synchronization failure final probability [3]. The item $(1 - \frac{1-G}{U+(1-G)})^j$ is the probability of precisely j transitions *PRESYNCH* → *HUNT* within a given cycle. The second item in (7.1) demonstrates that among δ consecutive cells there are exactly $\delta - j$ synchronous cells.

Remark 1. The discussion of the following problem is necessary in order to present a probabilistic model of the general CD process. The problem is related to the differentiation between synchronous and non-synchronous states. The *PRESYNCH* state status is, in essence, a disputable question. The relation to the *PRESYNCH* as the synchronous state as well as the non-synchronous state depends upon the researcher assumption. We assume that *PRESYNCH* state corresponds to the set of synchronous states of the CD process. In order to notify the Upper Layer Protocol of the ATM protocol stack that the system is in the "dangerous" and unstable mode it is worth to mark these cells. We suggest to use the reserved combination in the Payload Type (PLT) field in the cell Header. Thus, the ATM Adaptation Layer Protocol will be able to produce a complicated error recovery of the "pre-synchronous" cells.

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