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UNSTEADY SQUEEZING FLOW OF A VISCOUS MHD FLUID BETWEEN PARALLEL PLATES, A SOLUTION USING THE HOMOTOPY PERTURBATION METHOD

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Abstract. The present paper analyses the unsteady 2-dimensional flow of a viscous MHD fluid between two parallel infinite plates. The two infinite plates are considered to be approaching each other symmetrically, causing the squeezing flow. A similarity transformation is used to reduce the partial differential equations modeling the flow, to a single fourth-order non-linear differential equation containing the Reynolds number and the magnetic field strength as parameters. The velocity functions are obtained for a range of values of both parameters by using the homotopy perturbation method. The total resistance to the upper plate is presented.

Key words: MHD fluid, Homotopy Perturbation method, squeezing flow.

1 Introduction

The interaction of conducting fluids with electromagnetic fields is the well known area of Magento-Hydro Dynamics (MHD). The flow of a fluid that is under the influence of an electromagnetic field, *i.e.*, an MHD fluid between moving parallel plates leads to squeezing flow. Such a flow problem lends itself to applications in bearings with liquid-metal lubrications, for instance. The use of a MHD fluid as lubricant is of interest, because it prevents the unexpected variation of lubricant viscosity with temperature under certain extreme operating conditions. The MHD lubrication in an externally pressurized thrust bearing has been investigated both theoretically and experimentally by Maki *et al.* [16]. Other authors that have investigated the effects of a magnetic field in lubrication include [12] and [15] for instance. These authors had neglected the inertial terms in the Navier-Stokes equations. Hamza [4] considered the squeezing flow between two discs in the presence of a magnetic field. The problem of squeezing flow between rotating discs has been studied by [5] and later by [1].

In the present analysis we consider the 2-dimensional flow of an MHD fluid between parallel plates that are moving symmetrically about the line of axial symmetry, giving rise to the the squeezing flow. The approximate analytical solution to the equation is presented for an interesting useful class of squeezing flow in the presence of a magnetic field using the homotopy perturbation method. There are many different methods to solve nonlinear equations such as the artificial parameter method. Recently, He ([6, 7, 8, 9]) proposed a new perturbation method which is, in fact, a coupling of the traditional perturbation method [13, 14, 17] and homotopy as used in topology. This gives rise to the homotopy perturbation method (HPM). In several papers He applied this method to discuss non-linear boundary value problems ([6, 7, 8, 9]) as well as non-linear problems on bifurcation.

Due to the success of the homotopy perturbation method different researchers applied it to solve nonlinear differential equation in different field of applied mathematics. In fluid mechanics Siddiqui *et al.* [18, 19, 20, 21] used this method for solving non-linear problems involving Newtonian and non-Newtonian fluids. For a comprehensive account of the use of the HPM successfully to solve problems in fluid mechanics, please see [10, 11].

The objective of the paper is to apply the homotopy perturbation method to study the squeezing flow of an incompressible MHD fluid between two parallel plates. The unsteady Navier–Stokes equations, after employing a similarity transformation, reduce to a 4th order nonlinear ordinary differential equation involving the parameter R. In a previous study, for instance [1] solved a similar problem between rotating discs using a numerical method. Here, in this study, we decompose the nonlinear differential equation into linear and non linear parts, each involving the parameter R. The homotopy perturbation method does not pose any restrictions on the parameter R, i.e., it does not have to be small. We note that the authors have applied the method to solve a similar flow problem, where the fluid is not characterised as an MHD fluid.

The plan of the paper consists of Section 2, which develops the equations as well as boundary conditions governing the squeezing flow. Section 3 applies the homotopy perturbation method to obtain the solution of the problem. Section 4 deals with the total resistance to the upper plate. The results obtained are discussed graphically in Section 5. The special case of no magnetic field is shown, where the homotopy perturbation solution matches the results of [3], i.e., squeezing flow between parallel plates in the absence of a magnetic field.

2 Problem Formulation

We consider the rectilinear unsteady hydromagnetic squeezing flow of an incompressible two dimensional viscous fluid between two infinite parallel plates. The distance between the plates at any time t is $2a(t)$. The central axis of the channel is taken as the x-axis and the y-axis is normal to it. A uniform magnetic field $B = (0, B_0, 0)$ is acting along the y-axis and the induced magnetic field is assumed to be negligible. The magnetic field is of constant strength H_0 and it is applied in a direction perpendicular to the flow of the fluid. In fact $B_0 = H_0\mu_0$, where μ_0 is the magnetic permeability. It is also assumed that the plates move symmetrically with respect to the central axis of the channel (see, Fig. 1).

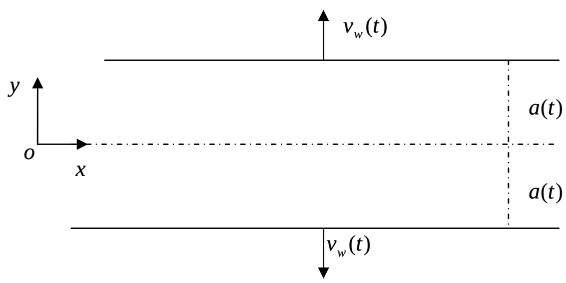

Figure 1. Rectilinear flow.

The unsteady mass and momentum conservation equations describing the flow are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{2.1}$$

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \sigma B_0^2 u, \tag{2.2}$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right), \tag{2.3}$$

where u and v are the velocity components along the x and y directions respectively, ν denotes the kinematic viscosity and ρ the density of the fluid, σ is the electrical conductivity of the fluid. We define the vorticity function ω and the generalized pressure h, respectively as

$$\omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}, \tag{2.4}$$

$$h = \frac{\rho}{2}(u^2 + v^2) + p. \tag{2.5}$$

On substituting (2.4) and (2.5) in (2.1), (2.2) and (2.3), the mass and momentum equations become

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{2.6}$$

$$\frac{\partial h}{\partial x}+\rho\left(\frac{\partial u}{\partial t}-v\omega\right)=-\nu\frac{\partial \omega}{\partial y}-\sigma B_0^2 u, \tag{2.7}$$

$$\frac{\partial h}{\partial y}+\rho\left(\frac{\partial v}{\partial t}+u\omega\right)=\nu\frac{\partial \omega}{\partial x}. \tag{2.8}$$

Eliminating the generalized pressure h using (2.7) and (2.8), we have a single momentum equation,

$$\rho\left(\frac{\partial \omega}{\partial t}+\left(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)\omega\right)=\nu\nabla^2\omega-\sigma B_0^2\frac{\partial u}{\partial y}. \tag{2.9}$$

The boundary conditions on $u(x,y,t)$ and $v(x,y,t)$ are

$$\text{at} \quad y=a, \quad u(x,y,t) = 0, \quad v(x,y,t)=v_w(t), \tag{2.10}$$

$$\text{at} \quad y=0, \quad v(x,y,t) = 0, \quad \frac{\partial u(x,y,t)}{\partial y}=0. \tag{2.11}$$

Here $v_w(t)=\dfrac{da}{dt}$ is the velocity of the plates. The first two conditions are due to the no-slip condition at the upper plate and the remaining two follow from the symmetry of the flow at $y=0$.

If the dimensionless variable $\eta = y/a(t)$ is introduced, where $2a(t)$ is the distance between the plates at any time, (2.6) and (2.9) become

$$\frac{\partial u}{\partial x}+\frac{\partial v}{a(t)\partial \eta}=0, \tag{2.12}$$

$$\rho\left(\frac{\partial \omega}{\partial t}+\left(u\frac{\partial}{\partial x}+v\frac{\partial}{a(t)\partial \eta}\right)\omega\right)=\nu\nabla^2\omega-\sigma B_0^2\frac{\partial u}{a(t)\partial \eta}. \tag{2.13}$$

The boundary conditions on $u(x,\eta,t)$ and $v(x,\eta,t)$ are

$$\text{at} \quad \eta=1, \quad u(x,\eta,t) = 0, \quad v(x,\eta,t)=v_w(t), \tag{2.14}$$

$$\text{at} \quad \eta=0, \quad v(x,\eta,t) = 0, \quad \frac{\partial u(x,\eta,t)}{\partial \eta}=0. \tag{2.15}$$

Let us define velocity components as ([2, 22])

$$u=\frac{C-x}{a(t)}v_w(t)f'(\eta), \quad v=v_w(t)f(\eta), \quad \omega=-\frac{C-x}{a(t)^2}v_w(t)f''(\eta), \tag{2.16}$$

where C is constant related to the inlet condition of the channel. By substituting (2.16) in (2.12) and (2.13), we find that the continuity equation is identically satisfied and (2.13) becomes

$$\frac{av_w}{\nu}\left(ff'''-f'f''-\eta f'''-2f''\right)+\frac{a^2}{\nu v_w}\frac{dv_w}{dt}f''=f^{iv}-f''M^2,$$

where $M^2 = \sigma B_0^2/\nu$ and the primes denote differentiation with respect to η. The boundary conditions are determined from (2.14),(2.15) and (2.16) to be

$$f(1) = 1, \quad f'(1) = 0, \tag{2.17}$$

$$f(0) = 0, \quad f''(0) = 0. \tag{2.18}$$

Thus for a similarity solution we define

$$\frac{av_w}{\nu} = R, \quad \frac{a^2}{\nu v_w}\frac{dv_w}{dt} = RQ, \tag{2.19}$$

where R and Q are both functions of t, but for a similarity solution R and Q are taken to be constants. After integrating the first equation of (2.19), we have

$$a(t) = (2\nu Rt + a_0^2)^{\frac{1}{2}}, \tag{2.20}$$

where $2a_0$ is the distance between the two plates at time $t = 0$. When $R > 0$, the plates move apart symmetrically with respect to $\eta = 0$ (or $y = 0$). In contrast,when $R < 0$, the plates approach each other and squeezing flow exists with similar velocity profiles as long as $a(t) > 0$. From (2.13) and (2.14), it follows that $Q = -1$, which means that (2.10) becomes

$$R\,(ff''' - f'f'' - \eta f''' - 3f'') = f^{iv} - M^2 f'', \tag{2.21}$$

subject to boundary condition (2.17) and (2.18).

3 Basic Idea of the Homotopy Perturbation Method

The homotopy perturbation method is a combination of the classical perturbation technique and the homotopy technique. To explain the basic idea of homotopy perturbation method, we consider the following non-linear differential equation

$$A(f) - f(\eta) = 0, \quad \eta \in \Omega, \tag{3.1}$$

subject to the boundary condition

$$C'\left(f, \frac{\partial f}{\partial \eta}\right) = 0, \quad \eta \in \partial\Omega,$$

where A is a general non-linear operator, C' is a boundary operator, $f(\eta)$ is known as an analytic function, $\partial\Omega$ is the boundary of the domain and $\dfrac{\partial f}{\partial \eta}$ is the directional derivative along the normal drawn outward from Ω.

The non-linear operator A, can be divided further into two parts: a linear part L and a non-linear part N. So that equation (3.1) can be written as

$$L(f) + N(f) - f(\eta) = 0.$$

By the homotopy technique, we construct a homotopy

$$f\,(\eta, q): \quad \Omega \times [0, 1] \to R,$$

which satisfies the equation

$$H\,[\,f,q\,] = (1-q)\,[\,L(f) - L(f_0)\,] + q\,[\,A(f) - f(\eta)\,] = 0,$$

which is equivalent to

$$H\,[\,f,q\,] = L(f) - L(f_0) + qL(f_0) + q\,[\,N(f) - f(\eta)\,] = 0, \tag{3.2}$$

where $q \in [0,1]$ is an embedding parameter, f_0 is the initial approximation of equation (3.2), which satisfies the boundary conditions. Therefore, we have

$$H(f,0) = L[\,f\,] - L[\,f_0\,] = 0, \;\; H(f,1) = A[\,f\,] - f(\eta) = 0.$$

Thus, the continuously changing q from zero to one is just that of changing $f(\eta,q)$ from $f_0(\eta)$ to $f(\eta)$. In topology, this kind of process is called a deformation, and $L(f) - L(f_0)$, $A(f) - f(\eta)$ are called homotopic.

In this method, we use the embedding parameter q as a small parameter and assume that the solution of (3.2) can be expanded as powers of q in terms of a series of the form,

$$f(\eta,q) = F_0 + qF_1 + q^2F_2 + \cdots\,, \tag{3.3}$$

and letting $q \to 1$, we note that (3.3) yields

$$\lim_{q\to 1} f(\eta,q) = F_0 + F_1 + F_2 + \cdots\,, \quad \text{or} \quad \lim_{q\to 1} f(\eta,q) = f\ (\eta).$$

The question of convergence of series in equation (3.3) has been discussed by He in [6].

4 Solution of the Problem Using the Homotopy Perturbation Method

As mentioned in the earlier section the HPM approach requires that we start by defining a homotopy $w\,(\eta,q):\ \Omega \times [0,1] \to R$ for (2.21) which satisfies the equation

$$L(w) - L(f_0) + qL(f_0) - Rq\left[w\frac{d^3w}{d\eta^3} - \frac{dw}{d\eta}\frac{d^2w}{d\eta^2} - \eta\frac{d^3w}{d\eta^3}\right] = 0, \tag{4.1}$$

where $L = \dfrac{d^4}{d\eta^4} + (3R - M^2)\dfrac{d^2}{d\eta^2}$ is the linear operator, $q \in [0,1]$ is the embedding parameter, f_0 is the initial guess approximation. We introduce $\theta = 3R - M^2$ to make the following solutions more compact. Let us take the initial guess approximation of (2.21) subject to boundary condition (2.17) and (2.18) as

$$f_0(\eta) = \left(\frac{\eta\sqrt{\theta}\cos\sqrt{\theta} - \sin\sqrt{\theta}\eta}{\sqrt{\theta}\cos\sqrt{\theta} - \sin\sqrt{\theta}}\right),$$

and the corresponding boundary conditions are given by

$$w(1) = 1, \quad \frac{dw}{d\eta}(1) = 0, \tag{4.2}$$

$$w(0) = 0, \quad \frac{d^2w}{d\eta^2}(0) = 0. \tag{4.3}$$

We assume that the solution of (2.21) can be expressed as a power series in q, i.e.,

$$w(\eta, q) = w_0 + qw_1 + q^2 w_2 + \cdots , \tag{4.4}$$

where the w_i are independent of q. Substituting (4.4) into (4.1), (4.2) and (4.3) and equating powers of q gives rise to a set of problems that we will specify and solve in the following sections.

4.1 The zeroth-order problem

The differential equation of the zeroth-order problem is

$$L[\, w_0 \,] - L[\, f_0 \,] = 0$$

under the boundary conditions

$$w_0(1) = 1, \quad w_0'(1) = 0,$$
$$w_0(0) = 0, \quad w_0''(0) = 0.$$

Since L is a linear operator, therefore the solution of the zeroth-order problem is

$$w_0(\eta) = \left(\frac{\eta\sqrt{\theta}\cos\sqrt{\theta} - \sin\sqrt{\theta}\eta}{\sqrt{\theta}\cos\sqrt{\theta} - \sin\sqrt{\theta}} \right) = f_0(\eta).$$

It can be shown that as $M \to 0$, $\theta \to 3R$, the non-magnetic zeroth order solution for w_0 is recovered. Furthermore, when we expand the trigonometric functions and let $R \to 0$, we recover the zeroth order solution of [3], i.e.,

$$w_0(\eta) = \frac{3\eta}{2} - \frac{\eta^3}{2} = f_0(\eta).$$

4.2 The first-order problem

The differential equation for the first-order problem is

$$L[\, w_1 \,] + L[\, f_0 \,] - R\,[w_0 w_0''' - w_0' w_0'' - \eta w_0'''] = 0$$

under the boundary conditions

$$w_1(1) = 1, \quad w_1'(1) = 0,$$
$$w_1(0) = 0, \quad w_1''(0) = 0.$$

The solution of the first-order boundary value problem is given by

$$
\begin{aligned}
w_1(\eta) = &-\epsilon\left(\frac{\eta\sqrt{\theta}\cos\sqrt{\theta}-\sin\sqrt{\theta}\eta}{\sqrt{\theta}\cos\sqrt{\theta}-\sin\sqrt{\theta}}\right)\\
&+\frac{\alpha}{4}\left(\frac{2\sin\sqrt{\theta}\eta}{(\theta)^{3/2}}-\frac{\eta\cos\sqrt{\theta}\eta}{\theta}-\frac{\eta^2\sin\sqrt{\theta}\eta}{\theta}\frac{4\eta\cos\sqrt{\theta}\eta}{(\theta)^{3/2}}+\frac{6\sin\sqrt{\theta}\eta}{\theta^2}\right)\\
&+\beta\left(\frac{\sin\sqrt{\theta}\eta}{\theta^2}-\frac{\eta\cos\sqrt{\theta}\eta}{2(\theta)^{3/2}}\right)-\gamma\eta,
\end{aligned}
$$

where α, β, γ and ϵ are all constants defined as

$$
\begin{aligned}
\alpha &= \frac{R(\theta)^{\frac{3}{2}}\sin\sqrt{\theta}}{(\sqrt{\theta}\cos\sqrt{\theta}-\sin\sqrt{\theta})^2}, \quad \beta = \frac{R(\theta)^{\frac{3}{2}}\cos\sqrt{\theta}}{(\sqrt{\theta}\cos\sqrt{\theta}-\sin\sqrt{\theta})^2},\\
\gamma &= \frac{\alpha}{4}\left(\frac{\sin\sqrt{\theta}}{\sqrt{\theta}}+\frac{\cos\sqrt{\theta}}{\theta}-\frac{\cos\sqrt{\theta}}{\sqrt{\theta}}+\frac{2\sin\sqrt{\theta}}{\theta}+\frac{2\cos\sqrt{\theta}}{(\theta)^{3/2}}\right)\\
&\quad+\frac{\beta}{2}\left(\frac{\sin\sqrt{\theta}}{\theta}-\frac{\cos\sqrt{\theta}}{(\theta)^{3/2}}\right),\\
\epsilon &= \frac{\alpha}{4}\left(\frac{2\sin\sqrt{\theta}}{(\theta)^{3/2}}-\frac{\cos\sqrt{\theta}}{\theta}-\frac{\sin\sqrt{\theta}}{\theta}-\frac{4\cos\sqrt{\theta}}{(\theta)^{3/2}}+\frac{6\sin\sqrt{\theta}}{\theta^2}\right)\\
&\quad+\beta\left(\frac{\sin\sqrt{\theta}}{\theta^2}-\frac{\cos\sqrt{\theta}}{2(\theta)^{3/2}}\right)-\gamma.
\end{aligned}
$$

It can be shown that as the magnetic field strength is reduced such that $B \to 0$, the non-magnetic first-order solution for w_1 is recovered. Since as $B \to 0$ we have that $M \to 0$, we note that this can happen also if $\sigma \to 0$, *i.e.,* the electrical conductivity of the fluid is very low or zero. But in both cases of either $B \to 0$ or $\sigma \to 0$ the fluid will loose its MHD character, reverting to a viscous fluid.

Furthermore, when we expand the trigonometric functions and let $R \to 0$, we recover the first order solution of [3], i.e.,

$$w_1(\eta) = \frac{\eta^5}{10} - \frac{\eta^7}{280} - \frac{53\eta^3}{280} + \frac{13\eta}{140}.$$

Finally, the homotopy perturbation solution of the problem up to the first order is

$$f(\eta) = \lim_{q\to 1} f(\eta, q) = w_0(\eta) + w_1(\eta) + \cdots,$$

or equivalently

$$f(\eta) = (1-\epsilon)\left(\frac{\eta\sqrt{\theta}\cos\sqrt{\theta} - \sin\sqrt{\theta}\eta}{\sqrt{\theta}\cos\sqrt{\theta} - \sin\sqrt{\theta}}\right)$$
$$+\frac{\alpha}{4}\left(\frac{2\sin\sqrt{\theta}\eta}{(\theta)^{3/2}} - \frac{\eta\cos\sqrt{\theta}\eta}{\theta} - \frac{\eta^2\sin\sqrt{\theta}\eta}{\theta} - \frac{4\eta\cos\sqrt{\theta}\eta}{(\theta)^{3/2}} + \frac{6\sin\sqrt{\theta}\eta}{\theta^2}\right)$$
$$+\beta\left(\frac{\sin\sqrt{\theta}\eta}{(\theta)^2} - \frac{\eta\cos\sqrt{\theta}\eta}{2(\theta)^{3/2}}\right) - \gamma\eta.$$

Now we compare our first order solution with [3]. On setting $M = 0$ we find that the non-magnetic solution at this order becomes

$$f(\eta) = (1-\epsilon)\left(\frac{\eta\sqrt{3R}\cos\sqrt{3R} - \sin\sqrt{3R}\eta}{\sqrt{3R}\cos\sqrt{3R} - \sin\sqrt{3R}}\right)$$
$$+\frac{\alpha}{4}\left(\frac{2\sin\sqrt{3R}\eta}{(3R)^{3/2}} - \frac{\eta\cos\sqrt{3R}\eta}{3R} - \frac{\eta^2\sin\sqrt{3R}\eta}{3R} - \frac{4\eta\cos\sqrt{3R}\eta}{(3R)^{3/2}} + \frac{6\sin\sqrt{3R}\eta}{(3R)^2}\right)$$
$$+\beta\left(\frac{\sin\sqrt{3R}\eta}{(3R)^2} - \frac{\eta\cos\sqrt{3R}\eta}{2(3R)^{3/2}}\right) - \gamma\eta,$$

and the constants α, β, γ and ϵ reduce to

$$\alpha = \frac{R(3R)^{\frac{3}{2}}\sin\sqrt{3R}}{(\sqrt{3R}\cos\sqrt{3R} - \sin\sqrt{3R})^2}, \quad \beta = \frac{R(3R)^{\frac{3}{2}}\cos\sqrt{3R}}{(\sqrt{3R}\cos\sqrt{3R} - \sin\sqrt{3R})^2},$$
$$\gamma = \frac{\alpha}{4}\left(\frac{\sin\sqrt{3R}}{\sqrt{3R}} + \frac{\cos\sqrt{3R}}{3R} - \frac{\cos\sqrt{3R}}{\sqrt{3R}} + \frac{2\sin\sqrt{3R}}{3R} + \frac{2\cos\sqrt{3R}}{(3R)^{3/2}}\right)$$
$$+\frac{\beta}{2}\left(\frac{\sin\sqrt{3R}}{3R} - \frac{\cos\sqrt{3R}}{(3R)^{3/2}}\right),$$
$$\epsilon = \frac{\alpha}{4}\left(\frac{2\sin\sqrt{3R}}{(3R)^{3/2}} - \frac{\cos\sqrt{3R}}{3R} - \frac{\sin\sqrt{3R}}{3R} - \frac{4\cos\sqrt{3R}}{(3R)^{3/2}} + \frac{6\sin\sqrt{3R}}{(3R)^2}\right)$$
$$+\beta\left(\frac{\sin\sqrt{3R}}{(3R)^2} - \frac{\cos\sqrt{3R}}{2(3R)^{3/2}}\right) - \gamma.$$

5 Results and Discussion

A similarity solution of the full Navier-Stokes equations for the unsteady flow between two parallel plates approaching or receding from each other under the influence of an electromagnetic field has been presented. It is shown that a similarity solution exists only when the distance between the plates varies as $(2\nu Rt + a_o^2)^{\frac{1}{2}}$, and squeezing flow takes place for $R > 0$. Employing He's homotopy perturbation method, we have obtained approximate analytical solutions for the fluid velocity for the flow of an unsteady 2-dimensional viscous MHD fluid between two parallel plates. Figure 2 shows that at a given time and for a fixed positive value of R the normal velocity increases monotonically from $\eta = 0$ to $\eta = 1$, for various values of the magnetic parameter M. Figure 3 shows that at a given time and for a fixed positive value of R, the longitudinal velocity decreases from $\eta = 0$ to $\eta = 1$ for various values of the magnetic parameter M. In addition, in Figure 4 we see that for a fixed positive value of the magnetic parameter M, the normal velocity increases monotonically from $\eta = 0$ to $\eta = 1$, for different values of R. Finally, in Figure 5 we see that for a fixed positive value of magnetic parameter M, the longitudinal velocity decreases monotonically from $\eta = 0$ to $\eta = 1$ for different values of R.

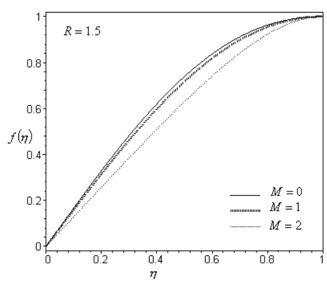

Figure 2. The normal velocity profiles for different values of magnetic parameter M, when $R = 1.5$ is fixed.

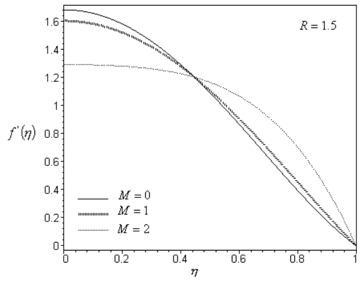

Figure 3. The longitudinal velocity profiles for different values of magnetic parameter M, when $R = 1.5$ is fixed.

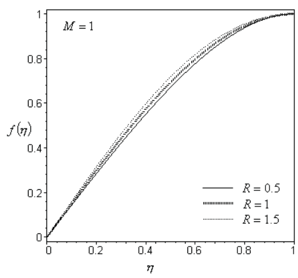

Figure 4. Shows the normal velocity profiles for different values of R, when $M = 1$ is fixed.

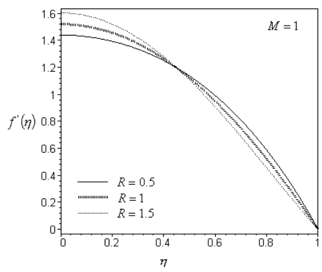

Figure 5. Shows the longitudinal velocity profiles for different values of R, when $M = 1$ is fixed.

In essence we have attained the solution of the problem and considered the special case in the absence of the magnetic field. In this case we found the solution to match the results of [3]. In addition, this paper demonstrates the effectiveness of the homotopy perturbation method, used for solving the full Navier-Stokes equations, describing the squeezing flow between two parallel plates. In contrast to the traditional perturbation technique, its initial approximation contains the whole linear portion involving the parameter R from the resulting non-linear differential equation. A significant advantage of the HPM is the freedom of selection of parameter R, it is not restricted to being small.

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