



BUCKLING LOAD ESTIMATION OF CRACKED COLUMNS USING ARTIFICIAL NEURAL NETWORK MODELING TECHNIQUE

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Abstract. In this paper, buckling analysis of slender prismatic columns with a single non-propagating open edge crack subjected to axial loads has been presented utilizing the transfer matrix method and the artificial neural networks. A multi-layer feedforward neural network learning by backpropagation algorithm has been employed in the study. The main focus of this work is the investigation of feasibility of using an artificial neural network to assess the critical buckling load of axially loaded compression rods. This is explored by comparing the performance of neural network models with the results of the matrix method for all considered support conditions. It can be seen from the results that the critical buckling load values obtained from the neural networks closely follow the values obtained from the matrix method for the whole data sets. The final results show that the proposed methodology may constitute an efficient tool for the estimation of elastic buckling loads of edge-cracked columns. Also, it can be seen from the results that the computational time reduces if the proposed method is used.

Keywords: buckling, stability, crack, slender prismatic columns, artificial neural networks.

1. Introduction

Columns and compression members are important structural elements used to carry and transfer axial or eccentric vertical loads in various engineering structures. In case of long columns and rods, the bearing capacity is usually limited and controlled by the buckling phenomenon. Buckling is defined as the change of equilibrium state of a column or rod from one configuration to another at a critical compressive load (Bazant, Cedolin 1991; Bazant 2000; Aristizabal-Ochoa 2004). The solutions for elastic buckling analysis of intact columns under various loading and boundary conditions are well documented (Bazant, Cedolin 1991; Timoshenko, Gere 1961; Wang *et al.* 2005). Columns and other structural elements may have real damages and weaknesses such as cracks. The cracks may develop from initial defects within the material, long-term service, impact, applied cyclic loads, etc. (Fan, Zheng 2003; Kishen, Kumar 2004). The presence of cracks changes the static and dynamic characteristics of the elements and structures, and cause failure at loads lower than their designed critical values (Jiki 2007). Therefore, the effects of cracks on the static and dynamic response of various structural elements are of great importance especially in structural, mechanical, earthquake, and aerospace engineering.

Many publications made on the stability of the cracked columns are available in the literature. Some

works are cited in the following without going into detail. Investigations on the stability and fracture of cracked columns started with the work of Liebowitz *et al.* (1967). They conducted experimental studies on notched and unnotched columns subjected to axial compressive loading. Okamura *et al.* (1969) studied the effects of reduced stiffness on the buckling of a slender column with a single edge crack. Anifantis and Dimarogonas (1983) developed a general flexibility matrix to study the stability of columns with a single edge crack subjected to follower and vertical loads. Nikipour (1990) studied the buckling of cracked composite columns. Takahashi (1999) performed the stability and vibration analyses of non-uniform Timoshenko beams using the transfer matrix method (TMM). Li (2001) studied the buckling of multi-step columns with arbitrary number of cracks including the shear deformation effects. Fan and Zheng (2003) investigated the effects of multiple cracks on the stability of Timoshenko beam-columns based on modified Fourier series. Kishen and Kumar (2004) studied the fracture behavior of eccentrically loaded cracked beam-columns using the finite element method. Gürel and Kısa (2005) investigated the buckling of slender prismatic columns with a single edge crack under concentric vertical load. Zhou and Huang (2006) investigated the effects of a single edge crack on the elastic buckling behavior of axially and eccentrically loaded rectangular cross-sectional columns. Gürel (2007) studied the buckling of slender prismatic circular cross-

sectional columns with multiple non-propagating edge cracks. Arboleda-Monsalve *et al.* (2007) presented stability and free vibration analyses of a crack-weakened Timoshenko beam-column with generalized end conditions subjected to constant axial load. Jiki (2007) performed a buckling analysis for the pre-cracked beam-columns using the Liapunov's second method. More recently, Goode *et al.* (2010) presented the analysis of the experimental data on concrete-filled steel tubes. Finally, Caddemi and Calio' (2008) have derived the exact solution of the uniform multi-cracked Euler-Bernoulli column, modeling by Dirac's deltas.

On the other hand, the use of artificial neural networks (ANNs) can be presented as an alternative method in estimating the critical buckling loads of columns and compression members. An ANN is a computational tool that attempts to simulate the architecture and internal operational features of human brain and neuron systems. Much of the success of ANN is due to its nonlinear and parallel processing characteristics. This technique has been applied in many disciplines including business, engineering, medicine and science (Inel 2007). In civil engineering, the neural networks has been successfully applied to a number of areas such as constitutive modeling (e.g. Ghaboussi *et al.* 1991), structural analysis and design (e.g. Inel 2007; Consolazio 2000), damage detection (e.g. Bilgehan 2011a; Suresh *et al.* 2004; Saridakis *et al.* 2008) structural dynamics and control (e.g. Chen *et al.* 1995) and non-destructive testing methods of material (e.g. Bilgehan, Turgut 2010a, b; Bilgehan 2011b; Hoła, Schabowicz 2005). To the knowledge of the authors, no work has been reported in the literature that addresses the application of the neural network approach for the estimation of critical buckling loads of cracked columns. This was the major motivation for the present study.

The purpose of this study to investigate the potential application of ANNs for the critical buckling load estimation of slender prismatic rectangular cross-sectional columns weakened by a single non-propagating edge crack. In this context, neural network approach is utilized for the critical buckling load estimation by using the non-dimensional crack depth and location values. The critical buckling load prediction is achieved through neural network models, which consist of two inputs, one hidden and one output layers.

The organization of the paper is as follows: Section 2 illustrates formulation of the matrix method for the buckling analysis of single cracked columns. The ANN method and architecture of the ANN model used in current study is explained in Section 3. In Section 4, application of the transfer matrix and neural network methods to the sample cracked columns and comparison of the results are presented. Finally, conclusions are presented in Section 5.

2. Transfer matrix method for the buckling analysis

The matrix method is an efficient and attractive tool for the solution of the eigenvalue problem for one-dimensional structures with non-uniform mechanical properties. The formulation follows the steps defined in Gürel and Kısı (2005) and Gürel (2007).

2.1. Problem formulation, governing equations and modal analysis

A slender prismatic column/compression rod with a rectangular cross section and having a non-propagating edge crack is shown in Fig. 1a. The end conditions of the column are not specified for making generalization. The coordinate system is chosen such that the x axis is along the length of the column and y axis is along the height of the cross section of the column. The crack is located at a distance of x_c from the upper end of the column. The physical model of the column is shown in Fig. 1b, in which the local flexibility due to the presence of the crack is considered. The cracked section is represented by a massless rotational spring with flexibility C , which is a function of the crack depth and height of the cross section of the column and can be written as (Shifrin, Ruotolo 1999):

$$C = 5.346hf(\xi), \quad (1)$$

where: h is the height of the column cross section and $\xi = ah$, where a is the depth of the crack, as presented in Fig. 1a. $f(\xi)$ is the dimensionless local flexibility computed from the strain energy function and is given by Shifrin and Ruotolo (1999) as:

$$f(\xi) = 1.8624\xi^2 - 3.95\xi^3 + 16.375\xi^4 - 37.226\xi^5 + 76.81\xi^6 - 126.9\xi^7 + 172\xi^8 - 143.97\xi^9 + 66.56\xi^{10}. \quad (2)$$

It can be seen from Fig. 1b that the column is divided into two segments, segment 1 ($0 \leq x \leq x_c$) and segment 2 ($x_c \leq x \leq L$), by the rotational spring.

The governing differential equation for buckling of segment 1 can be written as (Timoshenko, Gere 1961):

$$\frac{d^4 y_1}{dx^4} + k^2 \frac{d^2 y_1}{dx^2} = 0, \quad (3)$$

where: $k^2 = P/EI$, and P and EI are the axial compressive force and the flexural rigidity, respectively. In this case, the relationships among the displacement, slope, bending moment and shear force are:

$$\left. \begin{aligned} \theta_1(x) &= \frac{dy_1}{dx} \\ M_1(x) &= -EI \frac{d^2 y_1}{dx^2} \\ V_1(x) &= \frac{dM_1}{dx} - P \frac{dy_1}{dx} \end{aligned} \right\}. \quad (4)$$

The general solution of Eq. (3) is given by:

$$y_1(x) = A_1 + A_2 x + A_3 \sin(kx) + A_4 \cos(kx). \quad (5)$$

Using Eqs (4) and (5), the following relationship can be written:

$$\begin{Bmatrix} y_1(x) \\ \theta_1(x) \\ M_1(x) \\ V_1(x) \end{Bmatrix} = [B(x)] \begin{Bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{Bmatrix}, \quad (6)$$

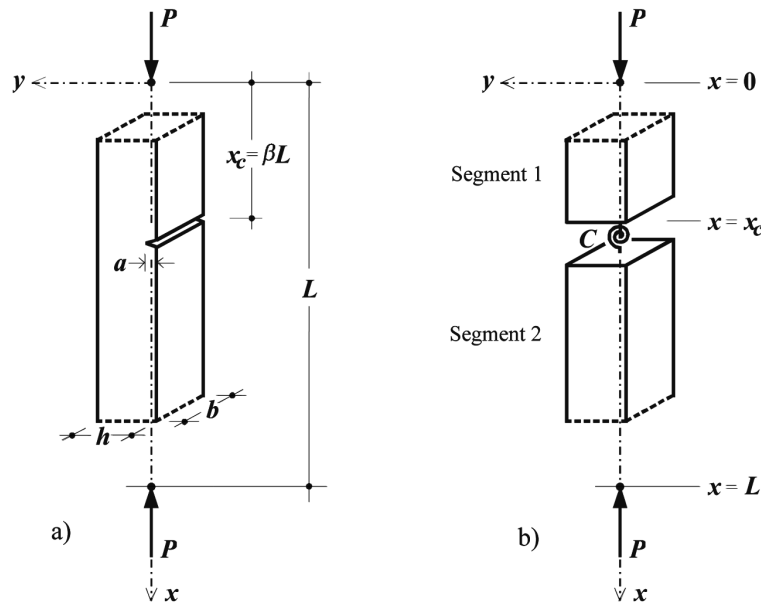


Fig. 1. A slender column with a non-propagating edge crack (a); its physical model (b)

where:

$$[B(x)] = \begin{bmatrix} 1 & x & \sin(x) & \cos(x) \\ 0 & 1 & k \cos(x) & -k \sin(kx) \\ 0 & 0 & P \sin(kx) & P \cos(kx) \\ 0 & -P & 0 & 0 \end{bmatrix} \quad (7)$$

The relationship between the parameters written above at the two ends of segment 1 can be expressed as:

$$\begin{Bmatrix} y_1(x_c) \\ \theta_1(x_c) \\ M_1(x_c) \\ V_1(x_c) \end{Bmatrix} = [T_1] \begin{Bmatrix} y_1(0) \\ \theta_1(0) \\ M_1(0) \\ V_1(0) \end{Bmatrix}, \quad (8)$$

in which:

$$[T_1] = [B(x_c)][B(0)]^{-1} \quad (9)$$

$[T_1]$ is called the transfer matrix for segment 1, because this matrix transfers the quantities at the upper end ($x = 0$) to those at the lower end ($x = x_c$) of segment 1. There is continuity among the displacements, bending moments, and shear forces at the common interface of segments 1 and 2, but there is a discontinuity between slopes at this point caused by the bending moment and rotation of the spring representing the cracked section. These conditions can be expressed as:

$$\left. \begin{aligned} y_1(x_c) &= y_2(x_c) \\ y_1''(x_c) &= y_2''(x_c) \\ y_1'''(x_c) &= y_2'''(x_c) \end{aligned} \right\}; \quad (10a)$$

$$\begin{aligned} \theta_2(x_c) - \theta_1(x_c) &= y_2'(x_c) - y_1'(x_c) = \Delta\theta(x_c) \\ &= Cy_1''(x_c) = -C \frac{M_1(x_c)}{EI} \end{aligned} \quad (10b)$$

Eq. (10b) is written by imposing equilibrium between the transmitted bending moment and the rotation of the spring.

Eqs (10a) and (10b) can be written in matrix form as:

$$\begin{Bmatrix} y_2(x_c) \\ \theta_2(x_c) \\ M_2(x_c) \\ V_2(x_c) \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -\frac{C}{EI} & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} y_1(x_c) \\ \theta_1(x_c) \\ M_1(x_c) \\ V_1(x_c) \end{Bmatrix} \quad (11)$$

Substitution of Eq. (8) into Eq. (11) yields:

$$\begin{Bmatrix} y_2(x_c) \\ \theta_2(x_c) \\ M_2(x_c) \\ V_2(x_c) \end{Bmatrix} = [T_{1C}] \begin{Bmatrix} y_1(0) \\ \theta_1(0) \\ M_1(0) \\ V_1(0) \end{Bmatrix}, \quad (12)$$

in which:

$$[T_{1C}] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -\frac{C}{EI} & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} [T_1] \quad (13)$$

The equation for segment 2 can be obtained by using Eqs (12) and (8):

$$\begin{Bmatrix} y_2(L) \\ \theta_2(L) \\ M_2(L) \\ V_2(L) \end{Bmatrix} = [T_2] \begin{Bmatrix} y_2(x_c) \\ \theta_2(x_c) \\ M_2(x_c) \\ V_2(x_c) \end{Bmatrix} = \quad (14)$$

$$[T_2][T_{1C}] \begin{Bmatrix} y_1(0) \\ \theta_1(0) \\ M_1(0) \\ V_1(0) \end{Bmatrix} = [T] \begin{Bmatrix} y_1(0) \\ \theta_1(0) \\ M_1(0) \\ V_1(0) \end{Bmatrix},$$

where:

$$[T] = [T_2][T_{1C}]. \quad (15)$$

The matrix $[T]$ has the following form:

$$[T] = \begin{bmatrix} T_{11} & T_{12} & T_{13} & T_{14} \\ T_{21} & T_{22} & T_{23} & T_{24} \\ T_{31} & T_{32} & T_{33} & T_{34} \\ T_{41} & T_{42} & T_{43} & T_{44} \end{bmatrix} \quad (16)$$

2.2. The eigenvalue equations and eigenvalues

The eigenvalue equations can be easily established imposing the boundary conditions on the Eq. (14). These are outlined in the following four columns with a variety of two end conditions.

(a) Fixed-free ended column: The boundary conditions for a fixed-free ended column are $M_1(0) = 0$, $V_1(0) = 0$ at $x = 0$ and $y_2(L) = 0$, $\theta_2(L) = 0$ at $x = L$. Therefore Eq. (14) becomes:

$$\begin{Bmatrix} 0 \\ 0 \\ M_2(L) \\ V_2(L) \end{Bmatrix} = \begin{bmatrix} T_{11} & T_{12} & T_{13} & T_{14} \\ T_{21} & T_{22} & T_{23} & T_{24} \\ T_{31} & T_{32} & T_{33} & T_{34} \\ T_{41} & T_{42} & T_{43} & T_{44} \end{bmatrix} \begin{Bmatrix} y_1(0) \\ \theta_1(0) \\ 0 \\ 0 \end{Bmatrix} \quad (17a)$$

Eq. (17a) reduces to the following form:

$$\begin{Bmatrix} 0 \\ 0 \end{Bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{Bmatrix} y_1(0) \\ \theta_1(0) \end{Bmatrix} \quad (17b)$$

For a nontrivial solution, by setting the determinant of the matrix in Eq. (17b) equal to zero, one obtains:

$$T_{11}T_{22} - T_{12}T_{21} = 0. \quad (18)$$

It can be seen that the eigenvalue equation is obtained from a 2×2 determinant.

Following the same procedure, the eigenvalue equations for other boundary conditions are obtained as:

(b) Pinned-pinned column:

$$T_{12}T_{34} - T_{14}T_{32} = 0; \quad (19)$$

(c) Fixed-pinned column:

$$T_{12}T_{24} - T_{14}T_{22} = 0; \quad (20)$$

(d) Fixed-fixed column:

$$T_{13}T_{24} - T_{14}T_{23} = 0. \quad (21)$$

After determining the elements T_{ij} of the matrix $[T]$ and then using Eqs (18), (19), (20) and (21), the eigenvalue equations are obtained in explicit form as:

(a) Fixed-free ended column:

$$\cos(kL) - Ck \sin(\beta kL) \cos[(1 - \beta)kL] = 0; \quad (22)$$

(b) Pinned-pinned column:

$$\sin(kL) - Ck \sin(\beta kL) \sin[(1 - \beta)kL] = 0; \quad (23)$$

(c) Fixed-pinned column:

$$[(kL) \cos(kL) - \sin(kL)] + Ck \sin(\beta kL) \{ \sin[(1 - \beta)kL] - (kL) \cos[(1 - \beta)kL] \} = 0; \quad (24)$$

(d) Fixed-fixed column:

$$4 \sin(kL/2) [\sin(kL/2) - (kL/2) \cos(kL/2)] + Ck \{ \sin(kL) - (kL) \cos(\beta kL) \cos[(1 - \beta)kL] \} = 0. \quad (25)$$

In Eqs (22) to (25) $\beta = x_c/L$ (Fig. 1a) and $k^2 = P/EI$ as stated earlier. By using a root-finder algorithm, the roots (eigenvalues) of the above equations can be obtained.

3. Artificial neural networks and application to the buckling analysis of the single cracked columns

A neural network is a computational structure inspired by the study of biological neural processing. There are many different types of neural networks, from relatively simple to very complex, just as there are many theories on how biological neural processing works. A layered feed-forward neural network, which is one of them, has layers, or subgroups of processing elements. A layer of processing elements makes independent computations on data that it receives and passes the results to another layer. The next layer may in turn make its independent computations and pass on the results to yet another layer. Finally, a subgroup of one or more processing elements determines the output from the network. Each processing element makes its computation based upon a weighted sum of its inputs. The first layer is the input layer and the last the output layer. The layers that are placed between the first and the last layers are the hidden layers. The processing elements are seen as units that are similar to the neurons in a human brain, and hence, they are referred to as cells, neuromimes, or artificial neurons. A threshold function is sometimes used to qualify the output of a neuron in the output layer. Even though our subject matter deals with artificial neurons, they will be simply referred to as neurons. Synapses between neurons are referred to as connections, which are represented by edges of a directed graph in which the nodes are the artificial neurons (V. B. Rao, H. Rao 1995).

The neural networks have ability in performing a good amount of generalization from the patterns on which they are trained. Training consists of exposing the neural network to a set of known input–output patterns. The data are passed through the multi-layered perceptron feedforward neural network in a forward direction only. As the data moves forward, it is subjected to simple processing within the neuron and along the links connecting neurons. The network performs successive iterations to adjust the weights of each neuron in order to obtain the target outputs according to a specific level of accuracy. The adjusting process of neuron weights is carried out to minimize the network error, which is defined as a difference between the computed and target output patterns. After the neural network is satisfactorily trained and tested, it is able to generalize rules and will be able to deal with unseen input data to predict output within the domain covered by the training patterns (Kartam *et al.* 1997; Rafiq *et al.* 2001; MathWorks 1999; Ashour, Alqedra 2005).

Among various network training methods, backpropagation is the most successful and widely used training algorithms for multi-linear perceptrons. In backpropagation, the input is propagated from the input layer through the hidden layers to the output layer. The calculated network error is then backpropagated from the output layer to the input layer. The aim of this process is to adjust weights so that the mean squared error is minimized. This process is repeated until the error is minimized to a preference level (Kartam *et al.* 1997; Flood, Kartam 1994). Simplified schema of an artificial neuron is shown in Fig. 2.

The generalized delta rule is a widely used learning mechanism in backpropagation neural networks (Rajagopalan *et al.* 1973). The implementation of such algorithm updates the network weights in the direction in which the performance function decreases most rapidly, reduces the total network error in the direction of the steepest descent of error (Kewalramani, Gupta 2006). The network consists of layers of parallel processing neuron elements with each layer being fully connected to the proceeding layer by interconnection strengths, or weights, W (Kisi 2005). Fig. 3 illustrates a three-layer neural network consisting of layers i , j and k ; input layer, hidden layer and output layer, respectively, with the interconnection weights W_{ij} and W_{jk} between layers of neurons. Initial estimated weight values are progressively corrected during a training process that compares predicted outputs with known outputs, and backpropagates any error to determine the appropriate weight adjustments necessary to minimize the errors. The typical multi-layer feedforward backpropagation neural networks are used for the buckling analysis of single cracked columns in the current study. The problem can be defined as a nonlinear input-output relation among the influencing factors which are the crack depth, crack location and the support conditions. An input vector consists of two components, namely, non-dimensional crack depth and non-dimensional crack location; and an output vector as the critical buckling load for each column.

The backpropagation algorithm and construction of the neural network model are carried out in the considered ANN simulation. Each batch of data is divided into two sets; one for the network learning, training set, and

the other for testing, testing set. The data set is normalized before the analyses and the predictive capabilities of the feedforward backpropagation neural network are examined. Neural network architecture used in this study is presented in Fig. 4.

4. Application of the matrix method and the neural networks model to the sample cracked columns/rods and comparison of the results

Four compression rods having fixed-free, pinned-pinned, fixed-pinned and fixed-fixed support conditions are considered to observe the effects of the crack depth ($a = \zeta h$) and the crack location ($x_c = \beta L$). The rods have the same cross-sectional dimensions of $h = b = 0.03$ m, and the same material with Young modulus of elasticity $E = 2 \times 10^4$ kN/cm², but different lengths of 0.65 m, 1.30 m, 1.85 m and 2.60 m, respectively. All rods buckle in the elastic range with these properties. The buckling load to the Euler load ratio (P_{cr}/P_E) versus crack location parameter ($\beta = x_c/L$) curves, corresponding to the 0.15, 0.35 and 0.50 values of the crack depth parameter ($\zeta = a/h$), are drawn and shown in Fig. 5 (a) to (d). It is evident from the figures that for all rods, when the crack depth and thus crack depth parameter increases, the buckling load and thus the P_{cr}/P_E ratio decreases. This is an expected result. The largest decrease is in the fixed-free ended rod, with a decrease of 20.50%, ($P_{cr}/P_E = 0.795$), and the smallest decrease occurs in the fixed-fixed rod, with a decrease of 5.84%, ($P_{cr}/P_E = 0.9416$).

Based on the support conditions, the location of crack has different effects. In a fixed-free ended rod, a crack at the fixed end causes the largest decrease in the buckling load, while in a pinned-pinned rod a crack located at mid-length has the largest effect for a constant crack depth. When the crack shifts towards any of the supports in a pinned-pinned rod, its effect diminishes. For fixed-pinned and fixed-fixed rods a crack at $x_c = 0.35 L$ and $x_c = 0.50 L$, respectively, causes the largest reduction in the buckling load. As it is well known from the fracture mechanics and the strength of materials, strain energy stored in an elastic body under a bending effect is directly related to the magnitude of the bending moments. Therefore, as the calculated results show, for all rod types, a crack located in the

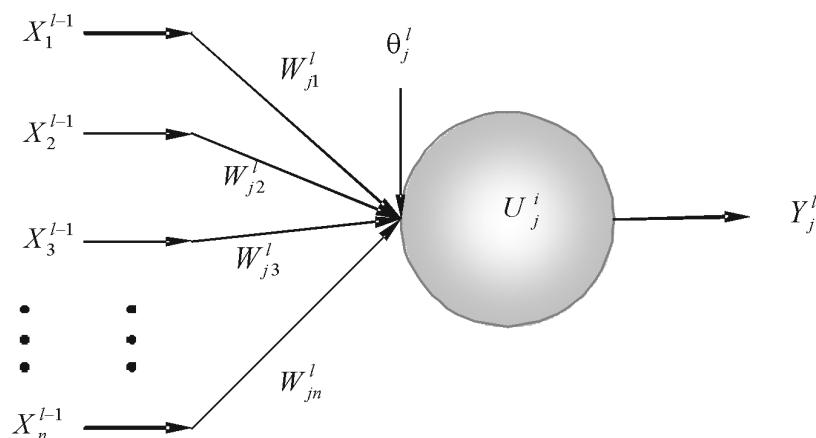


Fig. 2. Simplified schema of an artificial neuron

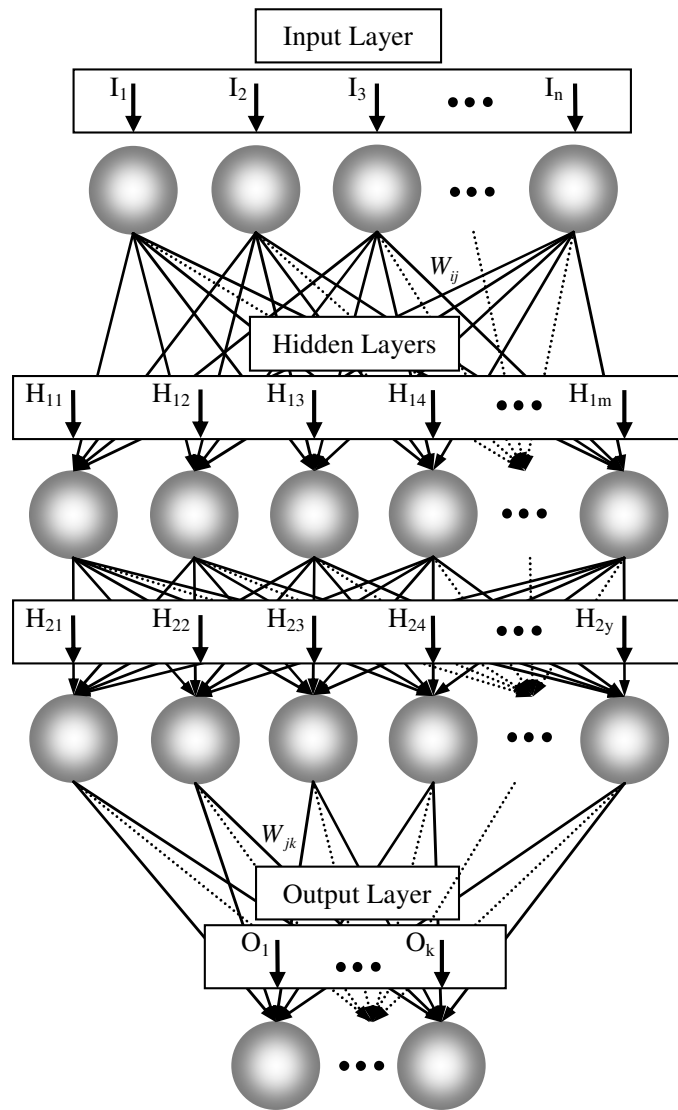


Fig. 3. Typical artificial neural network architecture

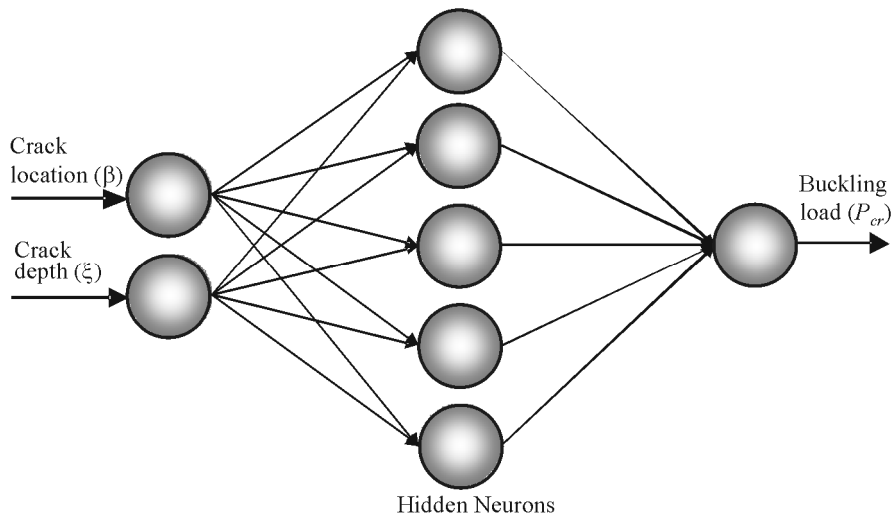


Fig. 4. Artificial neural network architecture used in this study

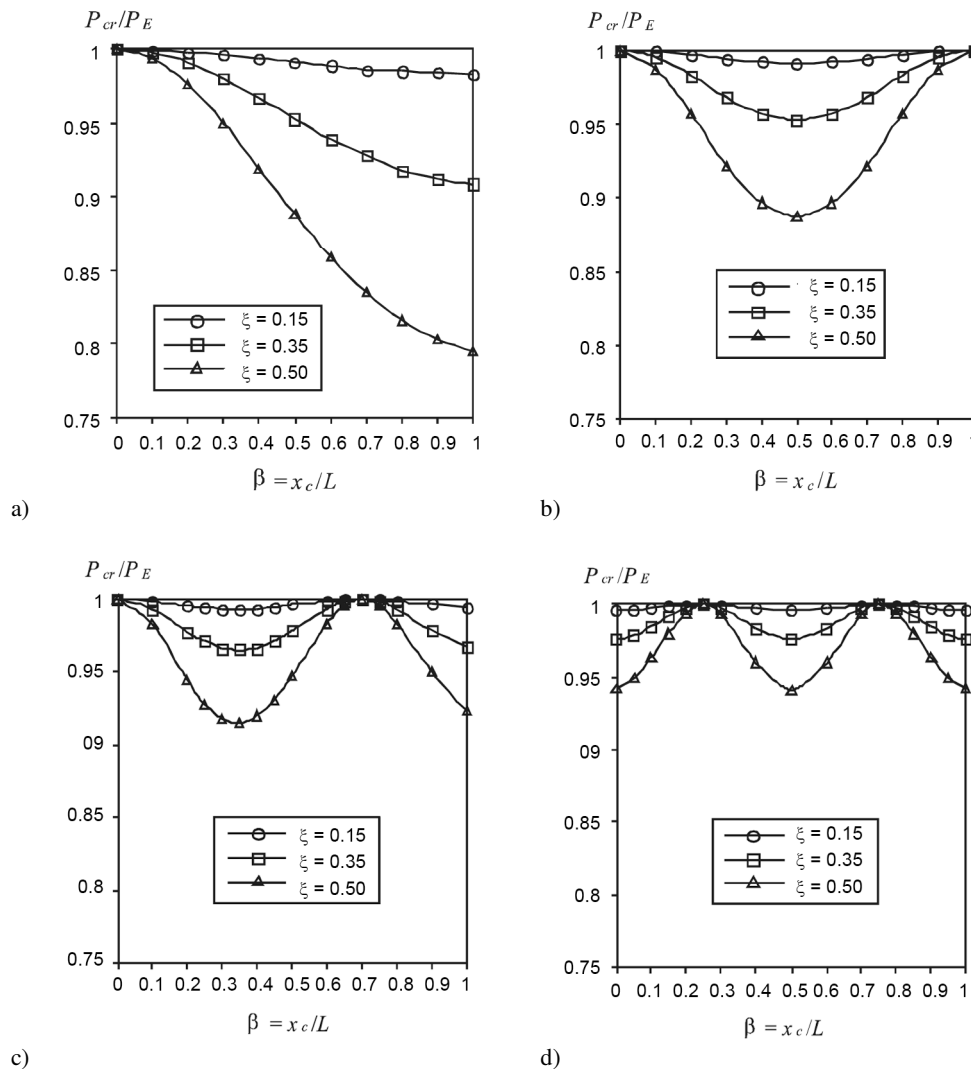


Fig. 5. Variation of the first critical buckling load to the Euler buckling load ratio (P_{cr}/P_E) depending on the dimensionless crack depth (ξ) and the dimensionless crack location (β): (a) fixed-free; (b) pinned-pinned; (c) fixed-pinned; and (d) fixed-fixed supported rods

section of maximum bending moments of the corresponding intact rods causes maximum energy losses and consequently the largest decrease in the buckling loads. Conversely, a crack located in the inflexion points, i.e. moment zero points of the corresponding intact rods has no effect on the critical buckling loads.

Buckling load values obtained with the matrix method for each rod are taken as training values for neural network models. For the considered fixed-free, pinned-pinned, fixed-pinned and fixed-fixed supported rods; 33, 33, 48 and 51 training buckling load values are used, respectively. Different number of the training data used for each support condition is chosen to perform in the best way of the training stage. Then, the trained ANN models tested with 6 testing data for each rod. The methodology used here for adjusting the weights is the momentum backpropagation with a delta rule, as presented by Rumelhart *et al.* (1986). Throughout all ANN simulations, the learning rates are used for increasing the convergence velocity. The computer program code, including neural networks toolbox, is available in MATLAB software.

Optimum hidden neuron numbers are obtained as five for all the considered support situations. The neural network toolbox needs some parameters to start simulation. Required parameter and its selected values are given in Table 1. In this table, the first row denotes the nodes of each layer for the neural network models. Accordingly, an ANN structure (2:5:1) consists of two inputs, five hidden and one output nodes. The range and statistical details of datasets are listed in Table 2.

Table 1. The final architecture of the ANN model in the elastic buckling analysis

ANN structure and parameters	
Number of nodes in layers	2:5:1
Training epoch number	5000
Momentum constant	0.9
Performance goal	0.00001
Learning rate	0.4

Table 2. The ranges and statistical details of the model variables

	Training Data			Testing Data		
	ξ	β	P_{cr} , kN	ξ	β	P_{cr} , kN
Minimum	0.15	0.00	65.8401	0.30	0.05	69.6293
Maximum	0.50	1.00	83.6613	0.45	1.00	82.8033
Average	0.33	0.50	80.8283	0.38	0.60	79.1705
Standard Dev.	0.14	0.31	3.2418	0.08	0.30	3.2422

The testing set is employed to evaluate the confidence in the performance of the trained network. The prediction performances are compared using two global statistics; the coefficient of determination (R^2) and the root mean squared error ($RMSE$), where the smaller the $RMSE$, the better are the estimates. $RMSE$ and R^2 values can be computed by the following standard formulas:

$$R^2 = 1 - \frac{\sum_{i=1}^N (A_i - P_i)^2}{\sum_{i=1}^N (A_i - \tilde{A}_i)^2}; \quad (26)$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^N (P_i - A_i)^2}{N}}, \quad (27)$$

where: P_i , A_i and \tilde{A}_i are the predicted, actual and averaged actual output of the network, respectively, and N is the total number of training patterns (Bilgehan 2011a, 2011b). The unit of measurement for $RMSE$ is kN.

The training performances of the neural network models in critical buckling load estimation for all support conditions are shown in Fig. 6(a)–(d). All data in these figures, falling to 1:1 line, indicates a successfully accomplished training phase.

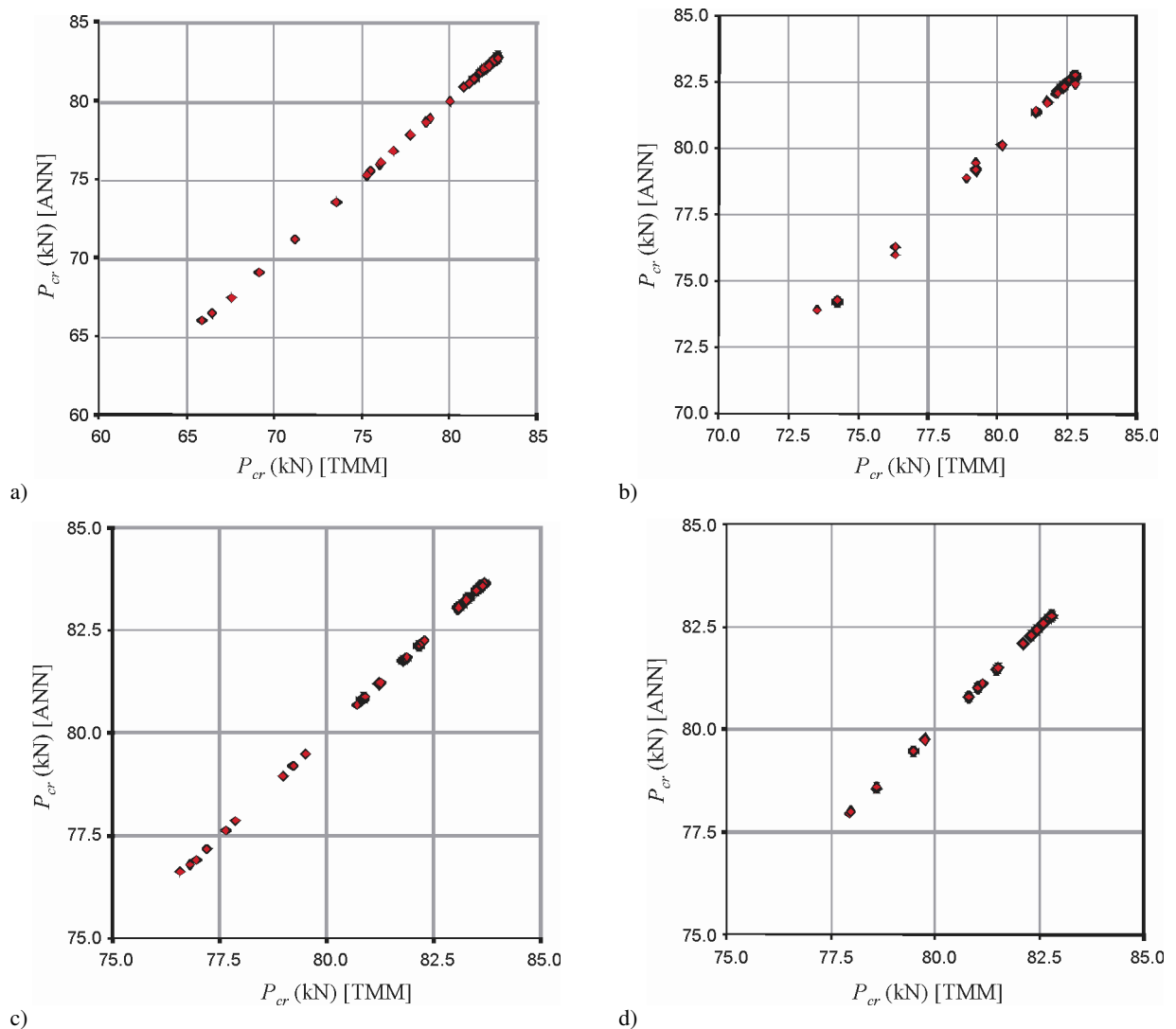


Fig. 6. The training performances of the models of: (a) fixed-free; (b) pinned-pinned; (c) fixed-pinned; and (d) fixed-fixed supported rods

Table 3. Comparison of performances of the models used based on statistical criteria in the critical buckling load estimation

Support condition of columns/rods under compression	Statistical criteria	
	R^2	$RMSE$
Fixed-free	0.957	1.25
Pinned-pinned	0.961	0.87
Fixed-pinned	0.982	0.47
Fixed-fixed	0.996	0.13

$RMSE$ and R^2 values of each model in the testing period are given in Table 3. According to the table, the $RMSE$ values range between 0.13 and 1.25; R^2 values range between 0.957 and 0.996. These are really narrow ranges. It can be seen from the table that the model with fixed-fixed support condition has the smallest $RMSE$ (0.13) and the highest R^2 (0.996). This presentation of error type is more realistic and meaningful. In this way, a more visual insight to the whole data set's performance can be obtained and analyzed.

The comparison of the critical buckling loads obtained from matrix method expressions (Eqs (22)–(25)) and estimated with neural network models are shown in Fig. 7. It can be seen from the graphs that the estimated values from neural networks closely follow the values from transfer matrix method. When the simulation results are further analyzed, it can be seen that the modeling

results are reasonably in good agreement with the matrix method solutions for data sets.

Computational time is another important criterion by which to compare performance of the models. Fig. 8 shows the comparison of the total projected time and computational time between the TMM and ANN method. Total time includes time of setup of the problem formulations and governing equations of the methods, writing of the related program codes, generating, training and testing of data, and so on. The computation is performed in the computer with Intel Core Duo processor with 2 GHz speed and 2.99 GB RAM. From Fig. 8, it can be seen that the neural network method can clearly reduce the computational time of about 65% (total projected time) and 81% (only computational time).

5. Conclusions and future works

In this paper buckling analysis of slender prismatic columns with a single non-propagating edge crack subjected to axial loads has been presented using the transfer matrix method and the neural networks. We have reported the details of a study on using multilayer feedforward ANN that learns by backpropagation algorithm for critical buckling load assessment for fixed-free, pinned-pinned, fixed-pinned and fixed-fixed supported, axially loaded compression rods.

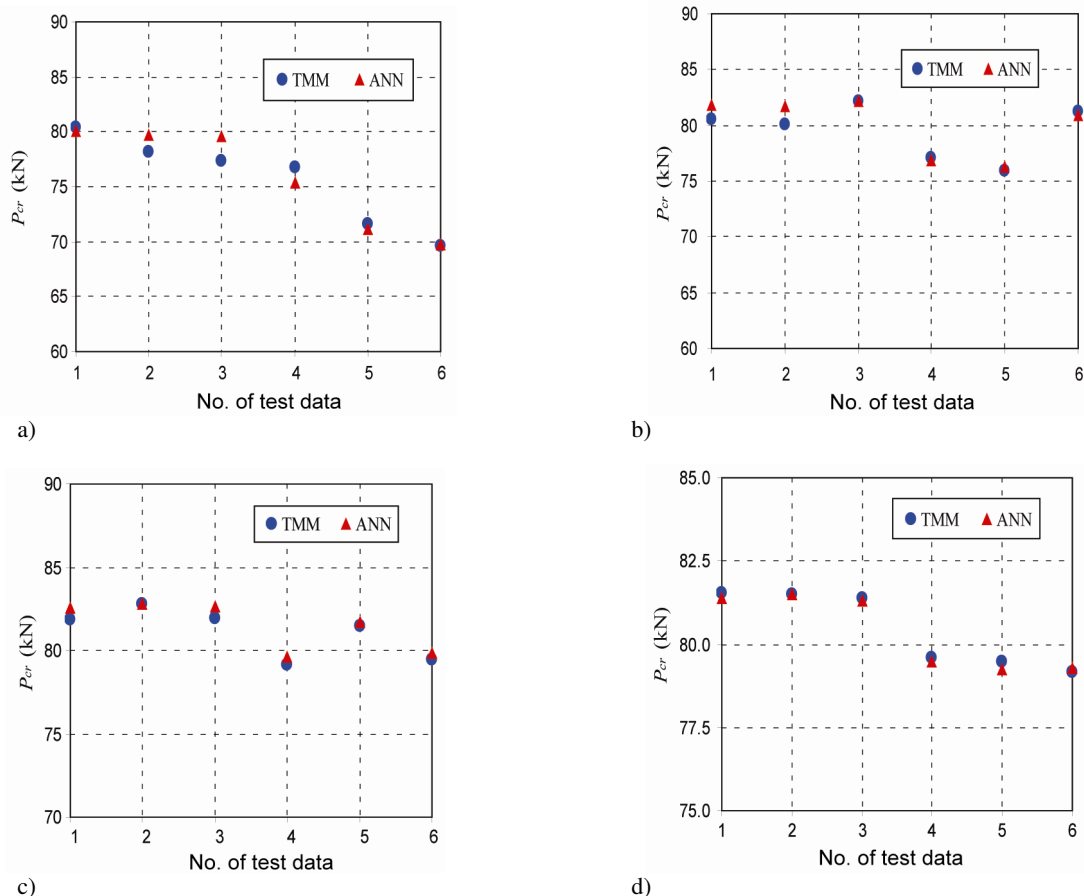


Fig. 7. Plotting of estimation performances for: (a) fixed-free; (b) pinned-pinned; (c) fixed-pinned; and (d) fixed-fixed supported rods

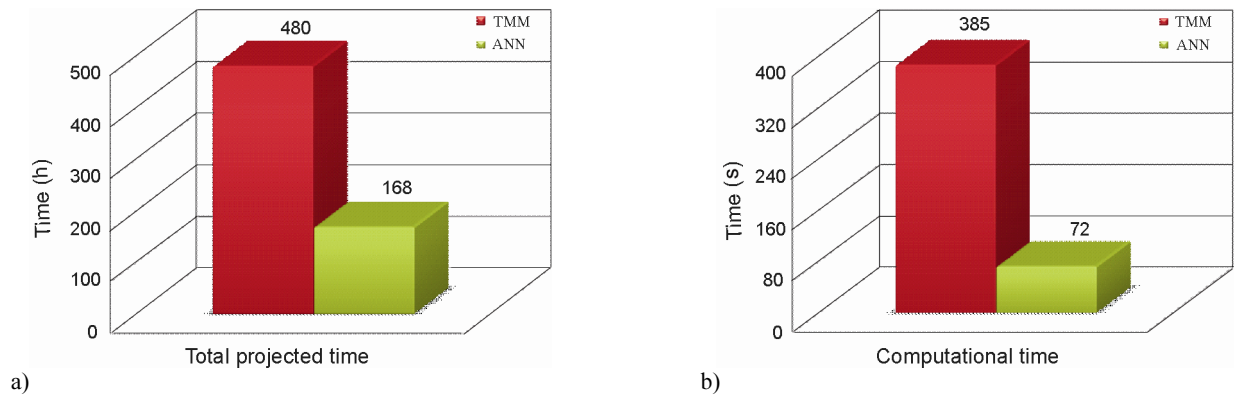


Fig. 8. Comparison of (a) total projected time and (b) computational time

The non-dimensional crack depth and the non-dimensional crack location values were taken as input data; critical buckling loads of the rods determined by transfer matrix method were taken as output data in training and testing phases of the neural network models. The constructed ANN model provides a quick and dependable means of predicting the buckling loads. An artificial neural network model with gradient descent algorithm having a hidden layer architecture established in the current study is seen suitable for this aim.

Some results are presented in the figures and tables to display both the combined effects of the boundary conditions, the depth and location of crack on the elastic buckling load and the efficiency of the neural network models. It can be seen from the figures and table that the neural network estimates closely follow the matrix method values. When the simulation results are further analyzed, it can be seen that the neural network modeling results are logically in good agreement with the transfer matrix solutions for whole data sets. This judgment has been supported by obtained values of RMSE and R^2 which are more realistic and meaningful error types. The transfer matrix method is a simple and efficient tool with which to analyze cracked columns. On the other hand, the buckling analysis of slender prismatic columns with TMM is required solutions of complex mathematical formulations. Nevertheless, the ANNs can be used effectively to solve mathematical problems with the complex formulation. Once the weights have been optimized, the appropriate results are obtained easily in seconds without any significant loss of accuracy when untried inputs are entered to the system. However, this study showed that the ANNs take less computational time without any significant loss of accuracy than the matrix method.

Although the analysis of the present study is mainly for neural network solutions of slender columns, an extension to stocky columns can be carried out easily by considering the effects of shear deformations. Other possible extensions of the study are the investigation of non-prismatic columns and the propagation and interaction of cracks studying with ANNs, which are left for future works. Furthermore, additional work must be conducted to improve efficiency of the techniques and it is hoped that other network topologies may lead to further improve performance.

Notations

The following notations are used in the present paper:

- β – non-dimensional crack location;
- ζ – non-dimensional crack depth;
- E – Young modulus of elasticity, kN/cm²;
- P_{cr} – critical buckling load, kN;
- P_E – Euler load, kN;
- P_i – predicted value;
- A_i – actual value;
- \tilde{A}_i – averaged actual value;
- N – number of data;
- U_j^l – net input of neuron j in layer l ;
- X_i^{l-1} – input coming from neuron i in layer $l-1$;
- Y_j^l – output of neuron j in layer l ;
- θ_j^l – threshold value;
- W_{ji}^l – weight between neuron j in layer l and neuron i in previous layer;
- R^2 – determination coefficient;
- h – height of the column cross section;
- a – depth of the crack, m;
- x_c – location of the crack, m;
- L – length of rod, m.

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