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HFMADM METHOD BASED ON NONDIMENSIONALIZATION AND ITS APPLICATION IN THE EVALUATION OF INCLUSIVE GROWTH

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Abstract. Inclusive growth, which encompasses different aspects of life, is a growth pattern that allows all people to participate in and contribute to growth process. In this paper, a novel hesitant fuzzy multiple attribute decision making (HFMADM) approach based on the nondimensionalization of decision making attributes is presented and then applied to the evaluation of inclusive growth in China. Firstly, a novel generalized hesitant fuzzy distance measure is proposed to calculate the difference and deviation between two hesitant fuzzy elements (HFEs) without adding any values into the shorter hesitant fuzzy element. Secondly, the coefficient of variation and efficacy coefficient method are extended to accommodate hesitant fuzzy environment and then used to cope with HFMADM. In the analysis process, non-dimensional treatment for hesitant fuzzy decision data is produced. Lastly, the method proposed in this paper is applied to an example of inclusive growth evaluation problem under hesitant fuzzy environment and the case study illustrates the practicality of the proposed method. Beyond that, a comparative analysis with some other approaches is also conducted to demonstrate the superiority and feasibility of the proposed method.

Keywords: hesitant fuzzy set, multiple attribute decision making, nondimensionalization, distance measure, coefficient of variation, efficacy coefficient, inclusive growth.

JEL Classification: C49, C61, D81.

Introduction

Decision-makers usually try to design policies, which support sustainable development (Collins *et al.* 2017). Since 2007, Asian Development Bank (ADB) and World Bank have successively put forward the concept of inclusive growth, which means that it would focus on high productivity growth that can lead to productive jobs, social inclusion that can ensure equality of opportunity, and a social safety net that can reduce risk and act as a cushion for the most vulnerable groups. Ali and Zhuang (2007) argue that inclusive growth aims to promote equal opportunities. McKinley (2010) builds the evaluation index system of inclusive growth based on standardized data and evaluates inclusive growth with respect to several attributes. Yu and Wang (2012) take the attribute weights into account and applied the proposed method to evaluate China's inclusive growth from 1990 to 2009. Collins *et al.* (2017) apply inclusive wealth theory to the evaluation of prospective policy. It is thus clear that inclusive growth can be expressed as a MADM problem. How to effectively evaluate inclusive growth seems to be an important and complicated task, which not only takes the evaluation attributes into account, but also should pay attention to the sustainable development of social economy.

In the real life, decision-makers often hesitate between several values to assess a variable or an alternative and no agreement is reached. In order to model this situation, Torra (2010) introduced the concept of hesitant fuzzy set, which can be considered as a generalization of fuzzy set (Zadeh 1965). It permits the membership value of an element to a set being represented by several possible values between 0 and 1. Since the HFS was proposed, it has been studied in depth by scholars (Rodríguez *et al.* 2016; Onar *et al.* 2016; Liu *et al.* 2015; Liu *et al.* 2016; Liao, Xu 2013; Alcantud *et al.* 2016; Ashtiani, Azgomi 2016; Wang *et al.* 2014; Li *et al.* 2015; Meng, Chen 2015; Zhu, Xu. 2014). Xia and Xu (2011) proposed a series of aggregation operators for hesitant fuzzy information and applied them to HFMADM problem. Wei (2012) presented the hesitant fuzzy prioritized operators, which can be used to deal with the decision problem with hesitant fuzzy information. Then, an intensive research on hesitant fuzzy information aggregation has been conducted (Zhang 2013; Yu *et al.* 2013; Zhu, Xu 2013; Qin *et al.* 2015; Peng, Wang 2014). Xu and Xia (2011a) proposed some hesitant fuzzy measures and defined the similarity measure between two HFSs, and then further research on the relationship between the distance, similarity measure and entropy of HFSs is conducted (Farhadinia 2013). However, these hesitant fuzzy distance measures cannot be calculated directly and several values need to be added into the shorter HFE. In order to overcome the drawbacks, Hu *et al.* (2016) and Peng *et al.* (2016) have successively put forward the generalized hesitant fuzzy distance measures, which can be calculated directly without adding any values into the shorter HFE, and then they applied them to HFMADM.

In the MADM problem with hesitant fuzzy information, there are two important issues that need to be addressed. Firstly, the attribute weights should be determined in advance. Secondly, an effective and appropriate decision method should be selected to aggregate the attribute value for each alternative and the optimal alternative can be obtained. As an important branch of fuzzy theory, Hesitant fuzzy multiple attribute decision making problem has attracted a lot of researchers' attention. However, few research efforts have aimed at how to avoid the impact of attribute magnitude and dimension, and most existing approaches to HFMADM consider little the significant influence of different evaluation attributes. Therefore, it is necessary to develop an approach to hesitant fuzzy decision making problem that can eliminate the effect of different physical dimensions on the final decision.

The remainder of this paper is arranged as follows. In Section 1, some preliminaries on HFSs are provided. Section 2 defines the coefficient of variation and efficacy coefficient under hesitant fuzzy environment, based on which an approach to hesitant fuzzy decision making problem is presented. In Section 3, an illustrative example is given to validate the effectiveness of the proposed method and a comparative analysis is also conducted. The last section ends the paper with some conclusions.

1. Preliminaries

As an extension of fuzzy set (Zadeh 1965), hesitant fuzzy set is very useful in handling the situation where people have hesitancy to evaluate an alternative or a variable. In what follows, we introduce some basic concepts related to hesitant fuzzy set.

Definition 1 (Torra 2010). Let X be a fixed set, then a hesitant fuzzy set on X is defined in terms of a function that when applied to X returns a subset of [0, 1].
To be understood easily, Xia and Xu (2011) utilized the following mathematical symbol to express hesitant fuzzy set:

$$E = \{\langle x, h_E(x)\rangle \,|\, x \in X\}, \tag{1}$$

where $h_E(x)$ is a set of several values in [0,1], denoting the possible membership degree of $x \in X$ to the set. For convenience, $h = h_E(x)$ is called a hesitant fuzzy element (HFE).

Based on the relationship between HFE and intuitionistic fuzzy value (IFV) (Atanassov 1986), Xia and Xu (2011) defined some new operations on HFEs.

Definition 2. Let h, h_1, h_2 be three HFEs, then
(1) $h^\lambda = \bigcup_{\gamma\in h}\{\gamma^\lambda\}$;
(2) $\lambda h = \bigcup_{\gamma\in h}\{1-(1-\gamma)^\lambda\}$;
(3) $h_1 \oplus h_2 = \bigcup_{\gamma_1\in h_1,\gamma_2\in h_2}\{\gamma_1+\gamma_2-\gamma_1\gamma_2\}$;
(4) $h_1 \otimes h_2 = \bigcup_{\gamma_1\in h_1,\gamma_2\in h_2}\{\gamma_1\gamma_2\}$.

In order to compare the HFEs, Xia and Xu (2011) gave the following comparison rule:

Definition 3. Let X be a fixed set. $h(x_i)=\left\{\gamma_{i1},\gamma_{i2},\cdots,\gamma_{il_i}\right\}$ is a HFE with $x_i \in X$. The score function of $h$is defined as

$$s(h(x_i))=\frac{1}{l_i}\sum_{k=1}^{l_i}\gamma_{ik}\ , \tag{2}$$

where l_i denotes the number of values in $h(x_i)$. For any two HFEs $h(x_1)$ and $h(x_2)$, if $s(h(x_1)) > s(h(x_2))$, then $h(x_1) > h(x_2)$; if $s(h(x_1)) = s(h(x_2))$, then $h_1 = h_2$.
However, in some cases, the comparison rule does not work. In order to address this issue, Chen *et al.* (2015) proposed a novel method that can be used to distinguish two HFEs.

Definition 4. Let X be a fixed set. $h(x_i)=\left\{\gamma_{i1},\gamma_{i2},\cdots,\gamma_{il_i}\right\}$ is a HFE with $x_i \in X$. The deviation function of $h(x_i)$ is defined as

$$v(h(x_i))=\sqrt{\frac{1}{l_i}\sum_{k=1}^{l_i}\left(\gamma_{ik}-s(h(x_i))\right)^2}\ , \tag{3}$$

where l_i denotes the number of values in $h(x_i)$. For any two HFEs $h(x_1)$ and $h(x_2)$, if $v(h(x_1)) > v(h(x_2))$, then $h(x_1) < h(x_2)$; if $v(h(x_1)) = v(h(x_2))$, then $h(x_1) = h(x_2)$.

Through the above analysis, we can find that the score function and deviation function are similar to the mean and variance in statistics, respectively. According to the score function and variance function above, a novel comparison rule for distinguishing two HFEs $h(x_1)$ and $h(x_2)$ can be obtained as follows:

If $s(h(x_1)) > s(h(x_2))$, then $h(x_1) > h(x_2)$;

If $s(h(x_1)) = s(h(x_2))$, then

if $v(h(x_1)) > v(h(x_2))$, then $h(x_1) < h(x_2)$;

If $v(h(x_1)) = v(h(x_2))$, then $h(x_1) = h(x_2)$.

Accordingly, the mean of HFS E can be obtained.

Definition 5 (Liao *et al.* 2015)**.** Assume that X is a fixed set. Let $E = \{\langle x_i, h_E(x_i)\rangle | x_i \in X\}$ be a HFS on X with $h_E(x_i) = \{\gamma_{Ei1}, \gamma_{Ei2}, \cdots, \gamma_{Eil_i}\}, i = 1, 2, \cdots, n$. The mean of HFS E is defined as follows:

$$s(E) = \frac{1}{n}\sum_{i=1}^{n} s(h_E(x_i)) = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{1}{l_{Ei}}\sum_{k=1}^{l_{Ei}}\gamma_{Eik}\right), \quad (4)$$

where l_{Ei} denotes the number of values in $h(x_i)$.

Given two HFEs $h(x_1)$ and $h(x_2)$, the number of values in different HFEs is usually different. Some researchers (Xu and Xia 2011a; Farhadinia 2013; Liao, Xu 2013; Wang *et al.* 2014; Li *et al.* 2015; Liu *et al.* 2015) suggest that the shorter HFE can be extended by adding any value in it until both of them have the same number. The value can be selected according to the decision-makers' risk preference.

Based on the extension rule above, Xu and Xia (2011b) put forward the following axioms for distance measure under hesitant fuzzy environment.

Definition 6. Let h_A and h_B be two HFEs on $X = \{x_1, x_2, \cdots, x_n\}$, then $d(h_A, h_B)$ denotes the distance measure between h_A and h_B, which satisfies the following properties:

(1) $0 \le d(h_A, h_B) \le 1$;

(2) $d(h_A, h_B) = 0$ if and only if $h_A = h_B$;

(3) $d(h_A, h_B) = d(h_B, h_A)$.

On the basis of Definition 6, Xu and Xia (2011b) gave the hesitant Euclidean distance for HFEs h_1 and h_2 as follows:

$$d(h_1, h_2) = \sqrt{\frac{1}{l_h}\sum_{i=1}^{l_h}\left|h_1^{\sigma(i)} - h_2^{\sigma(i)}\right|^2}, \quad (5)$$

where $h_1^{\sigma(i)}$ and $h_2^{\sigma(i)}$ are the ith largest values in HFEs h_1 and h_2, respectively, and $l_h = \max\{l_{h_1}, l_{h_2}\}$.

Drawing on traditional distance measures, such as Hamming distance, Euclidean distance and so on, Xu and Xia (2011a) define a variety of distance measures for HFSs. However, the extended distance measures for HFSs are calculated from HFEs with the same number of values. In other words, the shorter HFE should be extended by adding several values into it until the two HFEs are of equal length, which will inevitably affect the decision results. In order to overcome this drawback, Hu *et al.* (2016) and Peng *et al.* (2016) proposed novel distance measures for HFSs without adding any values into the shorter HFE, respectively. Definition 6 has some limitations. We first present a modification of it.

Definition 7 (Hu *et al.* 2016)**.** Let h_A and h_B be two HFEs. $h_A \prec h_B$ if and only if $\gamma_A < \gamma_B$ for any $\gamma_A \in h_A$ and $\gamma_B \in h_B$.

Definition 8 (Hu *et al.* 2016)**.** For three HFEs h_A, h_B and h_C, the distance measure between two HFEs, which is denoted as $d(\cdot\,,\cdot)$, satisfies the following properties:

(1) $0 \le d(h_A, h_B) \le 1$;

(2) $d(h_A, h_A) = 0$;

(3) $d(h_A, h_B) = d(h_B, h_A)$;

(4) if $h_A \prec h_B \prec h_C$, then $d(h_A, h_C) \ge d(h_A, h_B)$ and $d(h_A, h_C) \ge d(h_B, h_C)$.

Based on Definition 8, a generalized distance measure between two HFEs h_A and h_B is defined as follows (Hu *et al.* 2016):

$$d_1(h_A, h_B) = \left[\frac{1}{2}\left(\frac{1}{l_{h_A}}\sum_{\gamma_A \in h_A}\min_{\gamma_B \in h_B}\left|\gamma_A - \gamma_B\right|^{\lambda} + \frac{1}{l_{h_B}}\sum_{\gamma_B \in h_B}\min_{\gamma_A \in h_A}\left|\gamma_B - \gamma_A\right|^{\lambda}\right)\right]^{1/\lambda}, \quad (6)$$

where l_{h_A} and l_{h_B} denote the number of values in h_A and h_B respectively , and $\lambda > 0$.
Peng *et al.* (2016) also proposed a generalized distance measure based on Hausdorff distance between two HFEs h_A and h_B as follows:

$$d_2(h_A, h_B) = \left[\frac{1}{2}\left(\max_{\gamma_A \in h_A}\min_{\gamma_B \in h_B}\left|\gamma_A - \gamma_B\right|^{\lambda} + \max_{\gamma_B \in h_B}\min_{\gamma_A \in h_A}\left|\gamma_B - \gamma_A\right|^{\lambda}\right)\right]^{1/\lambda}. \quad (7)$$

The hesitant fuzzy distance measures above can be calculated directly from HFEs without having to add values into the shorter HFE. Therefore, it is more reasonable and suitable to solve the decision making problem with hesitant fuzzy information by using the distance measures above.

2. Hesitant fuzzy decision making method based on nondimensionalization

In multiple attribute decision making, direct aggregation of attribute values is sometimes impracticable in that there are different types of decision making attributes. For example, the cost attribute and benefit attribute cannot be aggregated directly. Since the physical dimensions or measurements of different decision making attributes are different, the attribute values should be normalized. In order to make the comprehensive evaluation value of each alternative comparable, the attribute values must be converted into a compatible scale, i.e., normalization or nondimensionalization. The aim of nondimensionalization is to eliminate the effect of different physical dimensions or measurements on the final decision. In this section, we present an approach based on nondimensionalization to solve HFMADM problem, in which the attribute weight information is unknown.

2.1. Problem description

In real life, individuals are often asked to select the optimal solution from a finite set of feasible alternatives against multiple attributes, which can be expressed as a MADM problem. This paper focuses on a MADM problem with hesitant fuzzy information.

For a HFMADM problem, we suppose that there are m alternatives $Y_i(i = 1, 2, \ldots, m)$ and n attributes $C_j(j = 1, 2, \ldots, n)$. Several experts or decision makers are invited to evaluate the alternatives with respect to each attribute. The evaluation value of alternative Y_i on attribute C_j takes the form of HFE h_{ij}. The alternative $Y_i(i = 1, 2, \ldots, m)$ and the attribute $C_j(j = 1, 2, \ldots, n)$ can be denoted by the vectors of HFS $Y_i = \left(h_{i1}, \quad h_{i2}, \quad \cdots, \quad h_{in}\right)$ and $C_j = \left(h_{1j}, \quad h_{2j}, \quad \cdots, \quad h_{nj}\right)^T$ respectively, where $h_{ij}(i = 1, 2, \cdots, m, j = 1, 2, \cdots, n)$ represents the attribute value of the ith alternative Y_i under the jth attribute. Assume that $w_j(j = 1, 2, \ldots, n)$ is the weight of the jth attribute $C_j(j = 1, 2, \ldots, n)$, which satisfies the following conditions:

$$\sum_{j=1}^{n} w_j = 1, 0 \le w_j \le 1(j = 1, 2, \cdots, n). \tag{8}$$

Therefore, the MADM problem with hesitant fuzzy information can be expressed in a hesitant fuzzy decision matrix as follows:

$$D = (h_{ij})_{m \times n} = \begin{matrix} \\ Y_1 \\ Y_2 \\ \vdots \\ Y_m \end{matrix} \begin{matrix} \begin{matrix} C_1 & C_2 & \cdots & C_n \end{matrix} \\ \begin{pmatrix} h_{11} & h_{12} & \cdots & h_{1n} \\ h_{21} & h_{22} & \cdots & h_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ h_{m1} & h_{m2} & \cdots & h_{mn} \end{pmatrix} \end{matrix}. \tag{9}$$

2.2. Coefficient of variation method for weight determination

In the MADM problem, attribute weight is one of the key factors that affect the decision results. In what follows, we adopt coefficient of variation method to determine the attribute weights for HFMADM problem. Coefficient of variation is a statistical measure that can be used to avoid the impact of different attribute magnitude and dimensions (Li *et al.* 2010). Generally, the greater the coefficient of variation of the attribute is, the greater the impact on the assessment is, and thus the attribute is more important.

Definition 9 (Liu 2016)**.** If Θ is a random variable with mean s and variance ν^2, then the parameter

$$\mu = \frac{\nu}{s}, \tag{10}$$

is called the coefficient of variation.

The coefficient of variation is a dimensionless number which can quantify the degree of variability relative to the mean. It is useful for people to use the coefficient of variation instead of the standard deviation when making a comparison between data sets with different means or different units.

Based on Definition 4, we first define the standard deviation of HFS E.

Definition 10. Assume that X is a fixed set. Let $E=\{\langle x_i, h_E(x_i)\rangle | x_i \in X\}$ be a HFS on X with $h_E(x_i)=\{\gamma_{Ei1},\gamma_{Ei2},\cdots,\gamma_{Eil_i}\}, i=1,2,\cdots,n$. The standard deviation of HFS E is defined as follows:

$$v(E)=\sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(s(h_E(x_i))-s(E)\right)^2}, \tag{11}$$

where $s(h_E(x_i))$ and $s(E)$ denote the score function of $h_E(x_i)$ and the mean of HFS E, respectively.

Let $w_j(j = 1, 2, \ldots, n)$ be the weight of the jth attribute $C_j(j = 1, 2, \ldots, n)$. In order to reflect the discrete degree of hesitant fuzzy evaluation value, we first calculate the coefficient of variation of each attribute as follows:

$$U(C_j)=\frac{v(C_j)}{s(C_j)}, j=1,2,\cdots,n, \tag{12}$$

where $v(C_j)$ and $s(C_j)$ represent the standard deviation and the mean of attribute C_j, respectively. They can be calculated by Eq. (11) and (4), respectively.

Then the attribute weight can be obtained by normalizing coefficient of variation of each attribute as follows:

$$w_j=\frac{U(C_j)}{\sum_{j=1}^{n}U(C_j)}, j=1,2,\cdots,n. \tag{13}$$

2.3. Hesitant fuzzy efficacy coefficient method

In this section, we extend the efficacy coefficient method to accommodate the situation in which the input arguments take the form of HFEs. The hesitant fuzzy efficacy coefficient method is then adopted to deal with the MADM problem with hesitant fuzzy information.

2.3.1. Novel hesitant fuzzy distance measure

Hu *et al.* (2016) and Peng *et al.* (2016) proposed novel distance measures for HFSs, respectively. They can be calculated directly from HFEs. However, the decision-makers' risk preference cannot be reflected and people do not know how to choose an appropriate hesitant fuzzy distance measure. Based on Definition 8, we first introduce a novel generalized hesitant fuzzy distance measure and discuss their relationship between them.

Definition 11. Let h_A and h_B be two HFEs. Then a generalized distance measure between two HFEs h_A and h_B can be defined as follows:

$$d_3(h_A,h_B)=\left[\frac{1}{2}\left(\sqrt[l_A]{\prod_{\gamma_A\in h_A}\min_{\gamma_B\in h_B}\left|\gamma_A-\gamma_B\right|^{\lambda}}+\sqrt[l_B]{\prod_{\gamma_B\in h_B}\min_{\gamma_A\in h_A}\left|\gamma_B-\gamma_A\right|^{\lambda}}\right)\right]^{1/\lambda}, \tag{14}$$

where $\lambda > 0$. Especially when $\lambda = 1$, the generalized distance measure $d_3(h_A, h_B)$ is reduced to the hesitant Hamming distance:

$$d_{hh}(h_A,h_B)=\frac{1}{2}\left(\sqrt[l_A]{\prod_{\gamma_A\in h_A}\min_{\gamma_B\in h_B}\left|\gamma_A-\gamma_B\right|}+\sqrt[l_B]{\prod_{\gamma_B\in h_B}\min_{\gamma_A\in h_A}\left|\gamma_B-\gamma_A\right|}\right). \quad (15)$$

When $\lambda = 2$, the generalized distance measure $d_3(h_A,h_B)$ is reduced to the hesitant Euclidean distance:

$$d_{he}(h_A,h_B)=\left[\frac{1}{2}\left(\sqrt[l_A]{\prod_{\gamma_A\in h_A}\min_{\gamma_B\in h_B}\left|\gamma_A-\gamma_B\right|^2}+\sqrt[l_B]{\prod_{\gamma_B\in h_B}\min_{\gamma_A\in h_A}\left|\gamma_B-\gamma_A\right|^2}\right)\right]^{1/2}. \quad (16)$$

In what follows, we prove that the generalized distance measure $d_3(h_A, h_B)$ satisfies the conditions (1)–(4) in Definition 8.

Proof: (1) It is straightforward.

(2) $$d_3(h_A,h_A)=\left[\frac{1}{2}\left(\sqrt[l_A]{\prod_{\gamma_A\in h_A}\min_{\gamma_A\in h_A}\left|\gamma_A-\gamma_A\right|^\lambda}+\sqrt[l_A]{\prod_{\gamma_A\in h_A}\min_{\gamma_A\in h_A}\left|\gamma_A-\gamma_A\right|^\lambda}\right)\right]^{1/\lambda}=0.$$

(3) $$d_3(h_A,h_B)=\left[\frac{1}{2}\left(\sqrt[l_A]{\prod_{\gamma_A\in h_A}\min_{\gamma_B\in h_B}\left|\gamma_A-\gamma_B\right|^\lambda}+\sqrt[l_B]{\prod_{\gamma_B\in h_B}\min_{\gamma_A\in h_A}\left|\gamma_B-\gamma_A\right|^\lambda}\right)\right]^{1/\lambda}=$$

$$\left[\frac{1}{2}\left(\sqrt[l_B]{\prod_{\gamma_B\in h_B}\min_{\gamma_A\in h_A}\left|\gamma_B-\gamma_A\right|^\lambda}+\sqrt[l_A]{\prod_{\gamma_A\in h_A}\min_{\gamma_B\in h_B}\left|\gamma_A-\gamma_B\right|^\lambda}\right)\right]^{1/\lambda}=$$

$$d_3(h_B,h_A).$$

(4) By Definition 7, if $h_A \prec h_B \prec h_C$, we can obtain that $\gamma_A \prec \gamma_B \prec \gamma_C$ for any $\gamma_A \in h_A$, $\gamma_B \in h_B$ and $\gamma_C \in h_C$. Therefore, $\left|\gamma_A-\gamma_C\right|>\left|\gamma_A-\gamma_B\right|$ and $\left|\gamma_A-\gamma_C\right|>\left|\gamma_B-\gamma_C\right|$, which imply that

$$d_3(h_A,h_C)=\left[\frac{1}{2}\left(\sqrt[l_A]{\prod_{\gamma_A\in h_A}\min_{\gamma_C\in h_C}\left|\gamma_A-\gamma_C\right|^\lambda}+\sqrt[l_C]{\prod_{\gamma_C\in h_C}\min_{\gamma_A\in h_A}\left|\gamma_C-\gamma_A\right|^\lambda}\right)\right]^{1/\lambda}\geq$$

$$\left[\frac{1}{2}\left(\sqrt[l_A]{\prod_{\gamma_A\in h_A}\min_{\gamma_B\in h_B}\left|\gamma_A-\gamma_B\right|^\lambda}+\sqrt[l_B]{\prod_{\gamma_B\in h_B}\min_{\gamma_A\in h_A}\left|\gamma_B-\gamma_A\right|^\lambda}\right)\right]^{1/\lambda}=$$

$$d_3(h_A,h_B),$$

and

$$d_3(h_A,h_C)=\left[\frac{1}{2}\left(\sqrt[l_A]{\prod_{\gamma_A\in h_A}\min_{\gamma_C\in h_C}\left|\gamma_A-\gamma_C\right|^\lambda}+\sqrt[l_C]{\prod_{\gamma_C\in h_C}\min_{\gamma_A\in h_A}\left|\gamma_C-\gamma_A\right|^\lambda}\right)\right]^{1/\lambda}\geq$$

$$\left[\frac{1}{2}\left(\sqrt[l_B]{\prod_{\gamma_B\in h_B}\min_{\gamma_C\in h_C}\left|\gamma_B-\gamma_C\right|^\lambda}+\sqrt[l_C]{\prod_{\gamma_C\in h_C}\min_{\gamma_B\in h_B}\left|\gamma_C-\gamma_B\right|^\lambda}\right)\right]^{1/\lambda}=$$

$$d_3(h_B,h_C).$$

The proof is completed. The parameter λ in the formula plays a part in the decision results. Each particular case of the proposed method may lead to different results. Decision-makers can select an appropriate one that is closest to her or his interests. Furthermore, the subjective information of attributes and the attitude of decision-makers can be reflected and it will also be able to provide more choices for decision-makers as the parameter λ changes.

Proposition 1. Let h_A and h_B be two HFEs. Then $d_3(h_A,h_B) < d_1(h_A,h_B) < d_2(h_A,h_B)$.

Proof: Since

$$\sqrt[n]{a_1 \cdot a_2 \cdot \cdots \cdot a_n} \le \frac{1}{n}\left(a_1 + a_2 + \cdots + a_n\right) \le \max\left\{a_1, a_2, \cdots, a_n\right\},$$

we can obtain

$$\sqrt[l_A]{\prod_{\gamma_A \in h_A} \min_{\gamma_B \in h_B} \left|\gamma_A - \gamma_B\right|^{\lambda}} + \sqrt[l_B]{\prod_{\gamma_B \in h_B} \min_{\gamma_A \in h_A} \left|\gamma_B - \gamma_A\right|^{\lambda}} \le$$

$$\frac{1}{l_{h_A}} \sum_{\gamma_A \in h_A} \min_{\gamma_B \in h_B} \left|\gamma_A - \gamma_B\right|^{\lambda} + \frac{1}{l_{h_B}} \sum_{\gamma_B \in h_B} \min_{\gamma_A \in h_A} \left|\gamma_B - \gamma_A\right|^{\lambda} \le$$

$$\max_{\gamma_A \in h_A} \min_{\gamma_B \in h_B} \left|\gamma_A - \gamma_B\right|^{\lambda} + \max_{\gamma_B \in h_B} \min_{\gamma_A \in h_A} \left|\gamma_B - \gamma_A\right|^{\lambda}.$$

Thus

$$\left[\frac{1}{2}\left(\sqrt[l_A]{\prod_{\gamma_A \in h_A} \min_{\gamma_B \in h_B} \left|\gamma_A - \gamma_B\right|^{\lambda}} + \sqrt[l_B]{\prod_{\gamma_B \in h_B} \min_{\gamma_A \in h_A} \left|\gamma_B - \gamma_A\right|^{\lambda}}\right)\right]^{1/\lambda} \le$$

$$\left[\frac{1}{2}\left(\frac{1}{l_{h_A}} \sum_{\gamma_A \in h_A} \min_{\gamma_B \in h_B} \left|\gamma_A - \gamma_B\right|^{\lambda} + \frac{1}{l_{h_B}} \sum_{\gamma_B \in h_B} \min_{\gamma_A \in h_A} \left|\gamma_B - \gamma_A\right|^{\lambda}\right)\right]^{1/\lambda} \le$$

$$\left[\frac{1}{2}\left(\max_{\gamma_A \in h_A} \min_{\gamma_B \in h_B} \left|\gamma_A - \gamma_B\right|^{\lambda} + \max_{\gamma_B \in h_B} \min_{\gamma_A \in h_A} \left|\gamma_B - \gamma_A\right|^{\lambda}\right)\right]^{1/\lambda},$$

which implies that

$$d_3(h_A,h_B) < d_1(h_A,h_B) < d_2(h_A,h_B).$$

The proof is completed.

Xu and Xia (2011b) have defined a variety of distance measures for HFEs. However, these distance measures are derived under the assumption that all the compared HFEs must be of equal length, and the shorter HFE would be extended by adding several values into it according to the decision-makers' risk preference. The proposed generalized hesitant fuzzy distance measure can be calculated directly from HFEs without adding any values into the shorter HFE. Furthermore, individual can also select a suitable distance measure according to his or her risk preference. If he or she is risk-seeking, the hesitant fuzzy distance measure $d_2(h_A, h_B)$ could fit in the situation. If he or she is risk-neutral, he or she can choose $d_1(h_A, h_B)$ for that situation. While he or she is risk-averse, $d_3(h_A, h_B)$ would be the most appropriate distance measure. In a word, the hesitant fuzzy distance measures $d_3(h_A, h_B)$, $d_1(h_A, h_B)$ and $d_2(h_A, h_B)$ can be calculated directly from HFEs without adding any values, and the decision-makers' risk preference can also be reflected.

2.3.2. Efficacy coefficient method for hesitant fuzzy decision problem

Efficacy coefficient method, which can reflect the complicated characteristic of multiple attributes, is a kind of quantitative decision analysis method and has been applied in many areas, such as the regional agricultural system (Yang, Gao 2006), geotechnical engineering (Wang *et al.* 2014), decision analysis (Wang, Z. S., Wang, L. J. 2011) and so on. It is based on the principle of multi-objective programming and can be used to eliminate influence of attribute magnitude and dimension.

Definition 12. Let $Y_i = (h_{i1},\ h_{i2},\ \cdots,\ h_{in})(i=1,2,\cdots,m)$ be an alternative, where $h_{ij}=\{\gamma_{i1},\gamma_{i2},\cdots,\gamma_{il_j}\}$ $(j=1,2,\cdots,n)$ is a HFE, then the vectors of hesitant fuzzy satisfaction value Y_i^+ and non-permissible value Y_i^- can be defined as follows, respectively:

$$Y_i^+ = (h_1^+, h_2^+, \cdots, h_n^+), \tag{17}$$

and

$$Y_i^- = (h_1^-, h_2^-, \cdots, h_n^-), \tag{18}$$

where

$$h_j^+ = \begin{cases} \max\limits_{1\le i\le m} \{\gamma_{i1},\gamma_{i2},\cdots,\gamma_{il_j}\}, & \text{for benefit attribute } C_j \\ \min\limits_{1\le i\le m} \{\gamma_{i1},\gamma_{i2},\cdots,\gamma_{il_j}\}, & \text{for cost attribute } C_j, \end{cases} \tag{19}$$

and

$$h_j^- = \begin{cases} \min\limits_{1\le i\le m} \{\gamma_{i1},\gamma_{i2},\cdots,\gamma_{il_j}\}, & \text{for benefit attribute } C_j \\ \max\limits_{1\le i\le m} \{\gamma_{i1},\gamma_{i2},\cdots,\gamma_{il_j}\}, & \text{for cost attribute } C_j. \end{cases} \tag{20}$$

In what follows, we extend the efficacy coefficient method to accommodate hesitant fuzzy environment.

Definition 13. Let Y_i^+ and Y_i^- be the vectors of hesitant fuzzy satisfaction value and non-permissible value, respectively. Then the efficacy coefficient of HFE h_{ij} can be obtained.

(1) For benefit attribute C_j, the efficacy coefficient of HFE h_{ij} is represented as follows:

$$E(h_{ij}) = \begin{cases} 1 & h_{ij} \ge h_j^+ \\ \dfrac{d(h_{ij}, h_j^-)}{d(h_j^+, h_j^-)} & h_j^- < h_{ij} < h_j^+ \\ 0 & h_{ij} \le h_j^- \end{cases}. \tag{21}$$

(2) For cost attribute C_j, the efficacy coefficient of HFE h_{ij} is represented as follows:

$$E(h_{ij}) = \begin{cases} 1 & h_{ij} \le h_j^+ \\ \dfrac{d(h_j^-, h_{ij})}{d(h_j^-, h_j^+)} & h_j^+ < h_{ij} < h_j^- \\ 0 & h_{ij} \ge h_j^- \end{cases}, \tag{22}$$

where $d(\cdot,\cdot)$ denotes the hesitant fuzzy distance measure between two HFEs.

Accordingly, the total efficacy coefficient of an alternative can be obtained as follows:

$$E(Y_i)=\sum_{j=1}^{n} w_j \cdot E(h_{ij}), \tag{23}$$

where $w_j(j = 1, 2, \ldots, n)$ is the weight of the jth attribute $C_j(j = 1, 2, \ldots, n)$.

Therefore, the greater the efficacy coefficient is, the better the comprehensive performance of an alternative is. People can select the optimal alternative according to the total efficacy coefficient.

2.4. An approach to hesitant fuzzy multiple attribute decision making

Based on the above analysis, a HFMADM method based on coefficient of variation and efficacy coefficient is presented, which is summarized as follows:

Step 1. For a hesitant fuzzy decision making problem, the decision-makers evaluate the alternatives $Y_i(i = 1, 2, \ldots, m)$ with respect to each attribute $C_j(j = 1, 2, \ldots, n)$, and the evaluation values take the form of HFEs. Thus, a hesitant fuzzy decision matrix $D=(h_{ij})_{m\times n}$, which is defined as shown in Eq. (9), can be constructed.

Step 2. Calculate the coefficient of variation of each attribute $C_j(j = 1, 2, \ldots, n)$ based on Eq. (12), and the attribute weight $w_j(j = 1, 2, \ldots, n)$ can be obtained by Eq. (13).

Step 3. Identify the vectors of hesitant fuzzy satisfaction value Y_i^+ and non-permissible value Y_i^- as shown in Eq. (17) and (18) respectively, and calculate each efficacy coefficient of HFE h_{ij} according to Eq. (21) and (22).

Step 4. Calculate the total efficacy coefficient of each alternative $Y_i(i = 1, 2, \ldots, m)$ by Eq. (23), and the optimal alternative can be obtained.

3. Illustrative example and comparative analysis

In this section, the evaluation of inclusive growth from McKinley (2010) and Yu and Wang (2012) is adopted to illustrate the proposed method. We apply the proposed method to the assessment of inclusive economy growth for Jiangsu Province in China, and the comparative analysis with some other methods is also conducted to demonstrate the effectiveness and superiority of the proposed method.

3.1. Illustrative example

In order to evaluate the inclusive growth level of Jiangsu Province in China, this study chooses six cities in the south, middle and north of Jiangsu Province, including Wuxi(Y_1), Suzhou(Y_2), Zhenjiang(Y_3), Nanjing(Y_4), Lianyungang(Y_5) and Xuzhou(Y_6) city (as shown in Fig. 1). An expert panel is invited to assess the inclusive growth level of the six cities with respect to seven attributes: C_1: sustainable economic growth; C_2: employment opportunity; C_3: income inequality; C_4: impoverishment rate; C_5: health and nutrition; C_6: education level; C_7: social security level. Among these attributes,

Fig. 1. Geographical position of six administrative regions in Jiangsu Province

C_3 and C_4 are cost attributes, and the rest are benefit attributes. As experts in different fields have diverse professional backgrounds, they may not reach an agreement and often hesitate among several values when evaluating an alternative with respect to an attribute. Therefore, HFS is a very useful tool to model this situation, and the experts prefer to adopt HFS to express their assessments.

The proposed method is utilized to evaluate the inclusive growth level of Jiangsu Province in China, and the following steps are involved:

Step 1. Experts from the expert panel assess the six cities with respect to seven attributes, where the assessments are in form of HFEs, and thus a hesitant fuzzy decision matrix $D = (h_{ij})_{6\times7}$ is constructed as listed in Table 1.

Table 1. Hesitant fuzzy decision matrix $D = (h_{ij})_{6\times7}$

	C_1	C_2	C_3	C_4	C_5	C_6	C_7
Wuxi	{0.6, 0.8}	{0.5, 0.7}	{0.4, 0.5}	{0.3, 0.5}	{0.6, 0.7}	{0.5}	{0.5, 0.8}
Suzhou	{0.7, 0.9}	{0.6, 0.7, 0.8}	{0.2, 0.3}	{0.2, 0.4, 0.5}	{0.7, 0.8}	{0.5, 0.7}	{0.7}
Zhenjiang	{0.5, 0.7}	{0.6}	{0.5, 0.6}	{0.5}	{0.6, 0.8}	{0.4, 0.6}	{0.6, 0.7}
Nanjing	{0.4, 0.6, 0.7}	{0.7, 0.8}	{0.5, 0.7}	{0.6}	{0.4, 0.6}	{0.8, 0.9}	{0.6, 0.8}
Lianyungang	{0.3, 0.4}	{0.4, 0.5}	{0.6, 0.7}	{0.5, 0.7}	{0.3, 0.5}	{0.1, 0.2}	{0.5}
Xuzhou	{0.4, 0.6}	{0.2, 0.4}	{0.7, 0.8}	{0.6, 0.8}	{0.2, 0.4}	{0.3, 0.5}	{0.4, 0.5}

Step 2. According to Eq. (12), we first calculate the coefficient of variation of each attribute $C_j (j = 1, 2, \ldots, 7)$, and then the attribute weight $w_j (j = 1, 2, \ldots, 7)$ can be obtained as follows:

$$W = \left(0.1273,\quad 0.1402,\quad 0.1550,\quad 0.1161,\quad 0.1561,\quad 0.2211,\quad 0.0842\right).$$

Step 3. By Eq. (17) and (18), the hesitant fuzzy satisfaction value Y_i^+ and non-permissible value Y_i^- can be determined:

$$Y_i^+ = \left(\{0.9\},\ \{0.8\},\ \{0.2\},\ \{0.2\},\ \{0.8\},\ \{0.9\},\ \{0.8\}\right),$$

$$Y_i^- = \left(\{0.3\},\ \{0.2\},\ \{0.8\},\ \{0.8\},\ \{0.2\},\ \{0.1\},\ \{0.4\}\right).$$

According to Eq. (21) and (22), each efficacy coefficient of HFE h_{ij} can be calculated as shown in the following matrix E:

$$E = \begin{bmatrix} 0.5723 & 0.5727 & 0.5387 & 0.5727 & 0.7060 & 0.5 & 0.375 \\ 0.7415 & 0.7443 & 0.8732 & 0.5967 & 0.8732 & 0.5561 & 0.75 \\ 0.4023 & 0.6667 & 0.3708 & 0.5 & 0.7415 & 0.4295 & 0.5563 \\ 0.2742 & 0.8732 & 0.2277 & 0.3333 & 0.4023 & 0.9053 & 0.6035 \\ 0 & 0.3708 & 0.2012 & 0.2277 & 0.2277 & 0 & 0.25 \\ 0.2277 & 0 & 0 & 0 & 0 & 0.3018 & 0 \end{bmatrix}.$$

Step 4. According to Eq. (23), the total efficacy coefficient of each city $Y_i(i = 1, 2, \ldots, 6)$ can be obtained:

$$E(Y_1) = 0.5555, E(Y_2) = 0.7258, E(Y_3) = 0.5178, E(Y_4) = 0.5451, E(Y_5) = 0.1662, E(Y_6) = 0.1179.$$

Therefore,

$$Y_2 > Y_1 > Y_4 > Y_3 > Y_5 > Y_6,$$

which implies that Suzhou boasts the highest potential for inclusive growth in Jiangsu Province.

Moreover, we can also find that Xuzhou, situated in the north of Jiangsu Province, has the lowest potential for inclusive growth. As a whole, cities in the south of Jiangsu Province rank well across the assessment attributes and have the highest potential for inclusive growth, while cities with medium-high inclusive growth potential are located in the middle part of Jiangsu Province.

As the parameter λ changes, we can get the rankings for inclusive growth of these cities, which are listed in Table 2. The results shown in Table 2 reveal that the rankings are unchanged with the change of the parameter λ. Also, the decision-makers can choose different values of the parameter λ according to their preferences.

Table 2. Results obtained by hesitant fuzzy distance $d_3(h_A, h_B)$

	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	Rankings
λ = 1	0.5555	0.7258	0.5178	0.5451	0.1662	0.0957	$Y_2 > Y_1 > Y_4 > Y_3 > Y_5 > Y_6$
λ = 2	0.5594	0.7286	0.5204	0.5508	0.1691	0.0977	$Y_2 > Y_1 > Y_4 > Y_3 > Y_5 > Y_6$
λ = 4	0.5662	0.7341	0.5254	0.5602	0.1741	0.1012	$Y_2 > Y_1 > Y_4 > Y_3 > Y_5 > Y_6$
λ = 8	0.5760	0.7436	0.5339	0.5716	0.1807	0.1060	$Y_2 > Y_1 > Y_4 > Y_3 > Y_5 > Y_6$
λ = 10	0.5796	0.7475	0.5372	0.5750	0.1827	0.1075	$Y_2 > Y_1 > Y_4 > Y_3 > Y_5 > Y_6$

3.2. Comparative analysis

In this section, a comparative analysis is conducted to demonstrate the superiority of the proposed method. We first adopt different hesitant fuzzy distance measures proposed by Hu *et al.* (2016) and Peng *et al.* (2016) to calculate the efficacy coefficient of each alternative according to the decision-makers' risk attitude.

3.2.1. Comparative analysis with hesitant fuzzy distance measures $d_1(h_A, h_B)$ and $d_2(h_A, h_B)$

If the decision-makers are risk-neutral, we adopt the hesitant fuzzy distance $d_1(h_A, h_B)$ to calculate the efficacy coefficient of each HFE h_{ij}, and the results are listed in Table 3. If the decision-makers are risk-seeking, the hesitant fuzzy distance $d_2(h_A, h_B)$ can be suited to calculate the efficacy coefficient of each HFE h_{ij}, and the results are listed in Table 4. From the results listed in Table 3 and 4, it can be seen that the optimal alternative is not subject to the effects of parameter setting, while the ranking orders are vulnerable to the impact of parameter changes. By comparison, we can find that the best alternative and optimal order are beyond the impact from the parameter uncertainties when adopting the proposed hesitant fuzzy distance $d_3(h_A, h_B)$. Certainly, the decision-maker can choose an appropriate distance measure to deal with the decision making problem according to his or her risk attitude and actual needs.

Table 3. Results obtained by hesitant fuzzy distance $d_1(h_A, h_B)$

	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	Rankings
$\lambda = 1$	0.5657	0.7313	0.5239	0.5571	0.1862	0.1470	$Y_2 > Y_1 > Y_4 > Y_3 > Y_5 > Y_6$
$\lambda = 2$	0.5828	0.7410	0.5340	0.5784	0.2118	0.1971	$Y_2 > Y_1 > Y_4 > Y_3 > Y_5 > Y_6$
$\lambda = 4$	0.6144	0.7623	0.5560	0.6167	0.2465	0.2574	$Y_2 > Y_4 > Y_1 > Y_3 > Y_6 > Y_5$
$\lambda = 8$	0.6567	0.8027	0.5929	0.6601	0.2799	0.3173	$Y_2 > Y_4 > Y_1 > Y_3 > Y_6 > Y_5$
$\lambda = 10$	0.6705	0.8183	0.6056	0.6729	0.2884	0.3262	$Y_2 > Y_4 > Y_1 > Y_3 > Y_6 > Y_5$

Table 4. Results obtained by hesitant fuzzy distance $d_2(h_A, h_B)$

	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	Rankings
$\lambda = 1$	0.6264	0.7965	0.5730	0.6204	0.2334	0.2175	$Y_2 > Y_1 > Y_4 > Y_3 > Y_5 > Y_6$
$\lambda = 2$	0.6447	0.8095	0.5848	0.6439	0.2575	0.2652	$Y_2 > Y_1 > Y_4 > Y_3 > Y_6 > Y_5$
$\lambda = 4$	0.6713	0.8321	0.6048	0.6750	0.2827	0.3031	$Y_2 > Y_4 > Y_1 > Y_3 > Y_6 > Y_5$
$\lambda = 8$	0.6990	0.8618	0.6298	0.7027	0.3026	0.3288	$Y_2 > Y_4 > Y_1 > Y_3 > Y_6 > Y_5$
$\lambda = 10$	0.7071	0.8712	0.6371	0.7098	0.3073	0.3346	$Y_2 > Y_4 > Y_1 > Y_3 > Y_6 > Y_5$

3.2.2. Comparative analysis with generalized hesitant weighted distance measure

Xu and Xia (2011a) presented a generalized hesitant weighted distance measure to solve hesitant fuzzy multiple attribute decision making problem, which is defined as follows:

$$d_{ghw}(M,N) = \left[\sum_{i=1}^{n} w_i \left(\frac{1}{l_{x_i}} \sum_{j=1}^{l_{x_i}} \left| h_M^{\sigma(j)}(x_i) - h_N^{\sigma(j)}(x_i) \right|^{\lambda} \right) \right]^{1/\lambda}, \quad (24)$$

where $h_M^{\sigma(j)}(x_i)$ and $h_N^{\sigma(j)}(x_i)$ are the jth largest values in HFEs $h_M(x_i)$ and $h_N(x_i)$, respectively. Then the distance between each alternative and the ideal alternative $A^* = \{1\}$ can be calculated and the rankings of the alternatives can be obtained. For the sake of comparison and analysis, we adopt the attribute weights obtained by the proposed method in Section 3.1:

$W = (0.1273,\ \ 0.1402,\ \ 0.1550,\ \ 0.1161,\ \ 0.1561,\ \ 0.2211,\ \ 0.0842)$.

Then the ranking results can be obtained, which are listed in Table 5. From the results as shown in Table 5, it can be seen that the optimal alternative is Nanjing (Y_4) city and the ranking results are also different from that obtained by the proposed method. The main reasons that account for the differences are in the following. Firstly, in their research, they do not differentiate between the cost and benefit attribute, and the impact of attribute magnitude and dimension is neglected. Secondly, the shorter HFE needs to be added several values into it when the generalized hesitant fuzzy distance $d_{ghw}(M, N)$ are adopted, which will affect the final decision. In our method, the hesitant fuzzy distance $d_3(h_A, h_B)$ can be directly calculated without considering the length of HFEs. Besides, the coefficient of variation and efficacy coefficient method can eliminate influence of different physical dimensions on the final decision. Therefore, the ranking results obtained by the proposed method are more reasonable.

Table 5. Results obtained by generalized hesitant fuzzy distance $d_{ghw}(M, N)$

	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	Rankings
λ = 1	0.4438	0.4100	0.4217	0.3551	0.5842	0.5236	$Y_4 > Y_2 > Y_3 > Y_1 > Y_6 > Y_5$
λ = 2	0.4624	0.4595	0.4342	0.3667	0.6124	0.5572	$Y_4 > Y_3 > Y_2 > Y_1 > Y_6 > Y_5$
λ = 4	0.4913	0.5400	0.4550	0.4144	0.6606	0.6046	$Y_4 > Y_3 > Y_1 > Y_2 > Y_6 > Y_5$
λ = 8	0.5337	0.6313	0.4870	0.4734	0.7267	0.6604	$Y_4 > Y_3 > Y_1 > Y_2 > Y_6 > Y_5$
λ = 10	0.5511	0.6573	0.4995	0.4922	0.7489	0.6787	$Y_4 > Y_3 > Y_1 > Y_2 > Y_6 > Y_5$

3.2.3. Comparative analysis with hesitant fuzzy TOPSIS method

Xu and Zhang (2013) extended the TOPSIS method to accommodate hesitant fuzzy environment. They presented an approach based on the maximizing deviation method and TOPSIS to tackle hesitant fuzzy decision making problem. According to their method, the shorter HFE is extended by adding the minimum value in it until the compared HFEs have the same length. Then the hesitant fuzzy decision matrix $\tilde{D} = (h_{ij})_{6\times 7}$ can be obtained as shown in Table 6.

Table 6. Hesitant fuzzy decision matrix $\tilde{D} = (h_{ij})_{6\times 7}$

	C_1	C_2	C_3	C_4	C_5	C_6	C_7
Wuxi	{0.6,0.6,0.8}	{0.5,0.5,0.7}	{0.4,0.4,0.5}	{0.3,0.3,0.5}	{0.6,0.6,0.7}	{0.5,0.5,0.5}	{0.5, 0.5,0.8}
Suzhou	{0.7,0.7,0.9}	{0.6,0.7,0.8}	{0.2,0.2,0.3}	{0.2,0.4,0.5}	{0.7,0.7,0.8}	{0.5,0.5,0.7}	{0.7,0.7,0.7}
Zhenjiang	{0.5,0.5,0.7}	{0.6,0.6,0.6}	{0.5,0.5,0.6}	{0.5,0.5,0.5}	{0.6,0.6,0.8}	{0.4,0.4,0.6}	{0.6,0.6,0.7}
Nanjing	{0.4,0.6,0.7}	{0.7,0.7,0.8}	{0.5,0.5,0.7}	{0.6,0.6,0.6}	{0.4,0.4,0.6}	{0.8,0.8,0.9}	{0.6,0.6,0.8}
Lianyungang	{0.3,0.3,0.4}	{0.4,0.4,0.5}	{0.6,0.6,0.7}	{0.5,0.5,0.7}	{0.3,0.3,0.5}	{0.1,0.1,0.2}	{0.5,0.5,0.5}
Xuzhou	{0.4,0.4,0.6}	{0.2,0.2,0.4}	{0.7,0.7,0.8}	{0.6,0.6,0.8}	{0.2,0.4,0.4}	{0.3,0.3,0.5}	{0.4,0.4,0.5}

On the other hand, the hesitant fuzzy positive ideal solution A^+ and negative ideal solution A^- can be determined as follows (Xu and Zhang 2013):

$$A^+ = \{\langle 0.7,0.7,0.9\rangle,\langle 0.7,0.7,0.8\rangle,\langle 0.7,0.7,0.8\rangle,\langle 0.6,0.6,0.8\rangle,\langle 0.7,0.7,0.8\rangle,\langle 0.8,0.8,0.9\rangle,\langle 0.7,0.7,0.8\rangle\},$$

$A^{-}=\{\langle 0.3,0.3,0.4\rangle,\langle 0.2,0.2,0.4\rangle,\langle 0.2,0.2,0.3\rangle,\langle 0.2,0.3,0.5\rangle,\langle 0.2,0.3,0.4\rangle,\langle 0.1,0.1,0.2\rangle,\langle 0.4,0.4,0.5\rangle\}$.

Based on the maximizing deviation method, we get the attribute weights as follows:

$W=(0.1345,\quad 0.1511,\quad 0.1449,\quad 0.1224,\quad 0.1478,\quad 0.1956,\quad 0.1037)$.

Based on the TOPSIS method, the relative closeness coefficient of each alternative $Y_i(i=1,2,\cdots,6)$ with respect to hesitant fuzzy positive ideal solution A^{+} can be obtained as follows:

$R(Y_1)=0.5578, R(Y_2)=0.6345, R(Y_3)=0.3989, R(Y_4)=0.7672, R(Y_5)=0.2783, R(Y_6)=0.3895$, which implies

$Y_4 > Y_2 > Y_1 > Y_3 > Y_6 > Y_5$.

Therefore, Nanjing (Y_4) city boasts the highest potential for inclusive growth in Jiangsu Province, which is different from that obtained by the method proposed in this paper. Furthermore, the ranking orders of the alternatives differ greatly from that obtained by the proposed method. The main reason lies in that the proposed method considers the difference between the cost and benefit attribute, which shall have significant effects on the stability of the final decision. The proposed method has several advantages over Xu and Zhang (2013) approach. On one hand, by extending the coefficient of variation and efficacy coefficient method to accommodate hesitant fuzzy environment, the proposed method can remove the impact of attribute magnitude and dimension. On the other hand, the proposed method utilizes the novel generalized hesitant fuzzy distance measure to calculate the difference and deviation between two HFEs without adding any values into the shorter HFE; while Xu and Zhang (2013) approach need the decision-makers to add several values into the HFE containing less value, which will affect the reliability of the decision results.

Beyond that, there are some approaches based on aggregation operators to HFMADM (Xia and Xu 2011; Wei 2012; Zhang 2013; Yu *et al.* 2013; Zhu, Xu 2013; Qin *et al.* 2015). Compared with those approaches, the method proposed in this paper can not only eliminate the impact of attribute magnitude and dimension, but also is robust to system variations caused by uncertain parameters.

Conclusions

Hesitant fuzzy set is a useful tool to model the situation where individuals have hesitancy to express their assessments. This paper presents an approach to HFMADM with unknown weights. A novel generalized hesitant fuzzy distance measure is proposed to calculate the difference and deviation between two HFEs without adding any values into the shorter HFE. People can choose an appropriate hesitant fuzzy distance measure to deal with decision making problem according to their risk attitude. To tackle the decision making problem with hesitant fuzzy information, we extend the coefficient of variation and efficacy coefficient method to accommodate hesitant fuzzy environment, which can avoid the impact of attribute magnitude and dimension. In addition, we have applied the proposed method to the evaluation of the inclusive growth level of Jiangsu Province in China and the comparative analysis with some other methods has been conducted. The comparison analysis shows that the proposed method not only can manage the hesitant fuzzy decision making with unknown weight information, but also can get a stable and reasonable decision result.

For real decision making problems, there are usually interactive characteristics for decision making attributes. However, the proposed approach only considers the importance of the given arguments, but ignore the correlation of decision making attributes. In future research, we expect to overcome this limitation. Besides, we will extend the method proposed in this paper to some other fuzzy areas, such as intuitionistic fuzzy set and dual hesitant fuzzy set, and utilize the proposed method to evaluate the sustainable development of ecological-economic system in China's Yangtze River Delta.

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