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# DELAMINATION DURING BUCKLING OF COMPOSITE CONSTRUCTIONAL ELEMENTS

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**Abstract.** Laminated constructional elements have widespread applications in aerospace. This paper presents of the buckling for E-Glass. Design solution based by the criteria of strength materials. The delamination of composite constructional elements is determined by the normal and shear stress. The non-linear strength criterion suggested by the author in case of complex state of stress.

**Keywords:** delamination, composite, strength, criterion.

## 1. Introduction

The delamination of composites depends on their matrix and the changed mechanical characteristics of reinforced elements during deformation. In the case of mechanical behaviour of laminated composites during compression the bending moment appears besides axial forces. The fibre experiences normal and shear stresses (Kim *et al.* 1998; Keller *et al*. 2004; Barbero *et al*. 1993; Barbero *et al.* 1999). Similar work was done while analysing the interfaces of I-beam shelves and walls (Bank *et al.* 1999), columns (Mosallam *et al.* 1992), beams (Bank *et al.* 1994), and cases of bar buckling depending on their geometry (Shu *et al.* 1993a; Shu *et al.* 1993b; Brewer *et al.* 1988). J. Brewer and P. Langace, as well as M. Fenske and A. Vizzini, suggested evaluation criteria for delamination. S. S.Wang and C. Hwu *et al.* tried solving the problems of composite fracture (Wang 1983; Hwu *et al.* 1995) The investigation of composite delamination remains topical, however, because the investigation and evaluation of fibre remain difficult.

## 2. Buckling of composite constructional elements

According to Euler's formula, critical buckling force is written as follows:

$$F_{cr} = \frac{4\pi^2 \left( EI_{ef} \right)}{L^2}, \tag{1}$$

where $F_{cr}$ – critical buckling force; $E$ – modulus of elasticity; $L$ – length of column; $I_{ef}$ – minimal moment of inertia.

The important characteristic of material is composite's modulus of elasticity $E_C$. It can be calculated in the following way (Bai *et al*. 2009):

$$E_C = \frac{2t_v E_v + 2t_m E_m + t_f E_f}{t} \tag{2}$$

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where $t$ – thickness of layer and indexes $v,m$ and $f$ mean cover, fiber and filling respectively.

The modulus of elasticity, $E_v$, is accepted as resin. The composite's $E_c$ is received experimentally.

Limit shear stresses $\tau_{\lim}$ are calculated in the following way (Bai *et al.* 2009)

$$\tau_{\lim} = \frac{1}{2}\sin 2\theta \cdot \sigma_U \tag{3}$$

where $\sigma_U$ – ultimate strength and $\theta$ – angle of layers with regard to stretching axis.

Lateral displacement is calculated as follows (Timoshenko *et al.* 1993)

$$w = \frac{w_{\max}}{2}\left(\cos\frac{2\pi x}{L} - 1\right) \tag{4}$$

where $w$ – lateral displacement; $x$ – coordinate in the longitudinal direction of the plate; $w_{\max}$ – maximal lateral displacement in the middle part of the plate during delamination.

Thus when the plate is compressed by force $F$, transverse forces $Q$ will be received in the following way (Timoshenko *et al.* 1993)

$$Q = F\sin\theta = F\sqrt{\frac{\tan^2\theta}{\tan^2\theta + 1}} \tag{5}$$

where

$$\tan\theta = \frac{dw}{dx}. \tag{6}$$

## 3. Strength criteria

In order to evaluate the strength of composites, various criteria are applied. One of the simplest is the Tresca criterion, which evaluates normal stresses and shear stresses (Fenske *et al.* 2001):

$$\frac{\sqrt{\sigma_x^2 + 4\tau_{xy}^2}}{2} \le \tau_{\lim}, \tag{7}$$

where $\sigma_x$ – normal stresses and $\tau_{xy}$ – shear stresses.

It has to be noted that normal stresses, $\sigma_y$, in the direction of *y* axis and shear stresses, $\tau_{yz}$, on the *yz* plane are quite small and need not be taken into account.

Then:

$$\tau_{xy} = \frac{dM}{dx}\frac{\int_A E(y)\,y\,dA}{bE_{eff}} = Q\frac{\int_A E(y)\,y\,dA}{bE_{eff}}, \tag{8}$$

where $A$ – area of cross-section; $M$ – bending moment.

The normal stresses are calculated in the following way:

$$\sigma_x = \frac{F\cos\theta}{A\left(V_r + nV_f\right)}, \tag{9}$$

where $V_r$ and $V_f$ are volumes of resin and reinforced elements, and $n$ is the ratio of elasticity module of reinforcement and matrix.

According to the yield criterion of von Mises (Bai *et al.* 2009)

$$\sqrt{\sigma_x^2 + 3\tau_{xy}^2} = \sigma_Y, \tag{10}$$

where $\sigma_Y$ – yield stress or

$$\frac{\sqrt{\sigma_x^2 + 3\tau_{xy}^2}}{2} = \frac{\sigma_Y}{2} = \tau_{\lim}. \tag{11}$$

The main stresses are calculated in the following way:

$$\begin{aligned}\sigma_1 &= \frac{1}{2}\left(\sigma_x + \sqrt{\sigma_x^2 + 4\tau_{xy}^2}\right)\\ \sigma_2 &= \frac{1}{2}\left(\sigma_x - \sqrt{\sigma_x^2 + 4\tau_{xy}^2}\right)\end{aligned}. \tag{12}$$

We shall apply polynomial strength criteria (Vasiliev *et al.* 2007):

$$F\left(\sigma_1, \sigma_2, \tau_{12}\right) = R_{11}\sigma_1^2 + R_{22}\sigma_2^2 + S_{12}\tau_{12}^2 = 1, \tag{13}$$

where $R$ and $S$ – constants

$$\tau_{12} = \frac{\sigma_1 - \sigma_2}{2}.$$

We find the constants $R$ and $S$ from the marginal conditions:

$$\begin{aligned}&F\left(\sigma_1 = \bar{\sigma}_1,\ \sigma_2 = 0,\ \tau_{12} = 0\right) = 1\\ &F\left(\sigma_1 = 0,\quad \sigma_2 = \bar{\sigma}_2,\ \tau_{12} = 0\right) = 1\\ &F\left(\sigma_1 = 0,\ \sigma_2 = 0,\ \tau_{12} = \bar{\tau}_{12}\right) = 1\end{aligned} \tag{14}$$

Then equation (13) is as follows:

$$\left(\frac{\sigma_1}{\bar{\sigma}_1}\right)^2 + \left(\frac{\sigma_2}{\bar{\sigma}_2}\right)^2 + \left(\frac{\tau_{12}}{\bar{\tau}_{12}}\right)^2 = 1. \tag{15}$$

The stresses $\bar{\sigma}_1, \bar{\sigma}_2$ are received:

$$\begin{aligned}&\bar{\sigma}_1 = \bar{\sigma}_1^+, \text{ if } \sigma_1 > 0 \text{ or } \bar{\sigma}_1 = \bar{\sigma}_1^-, \text{ if } \sigma_1 < 0\\ &\bar{\sigma}_2 = \bar{\sigma}_2^+, \text{ if } \sigma_2 > 0 \text{ or } \bar{\sigma}_2 = \bar{\sigma}_2^-, \text{if } \sigma_2 < 0\end{aligned} \tag{16}$$

When the strength criterion is put in this form (Vasiliev *et al.* 2007):

$$F\left(\sigma_1,\sigma_2,\tau_{12}\right)=R_1\sigma_1+R_2\sigma_2+R_{11}\sigma_1^2+ \\ +R_{22}^2\sigma_2^2+S_{12}\tau_{12}^2=1 \tag{17}$$

the marginal conditions:

$$\begin{aligned} &F\left(\sigma_1=\bar{\sigma}_1^+,\sigma_2=0,\,\tau_{12}=0\right)=1,\text{ if }\sigma_1>0\\ &F\left(\sigma_1=-\bar{\sigma}_1^-,\sigma_2=0,\,\tau_{12}=0\right)=1,\text{if }\sigma_1<0\\ &F\left(\sigma_1=0,\sigma_2=\bar{\sigma}_2^+,\,\tau_{12}=0\right)=1,\text{ iif }\sigma_2<0\\ &F\left(\sigma_1=0,\sigma_2=-\bar{\sigma}_2^-,\,\tau_{12}=0\right)=1,\text{if }\sigma_2<0\\ &F\left(\sigma_1=0,\sigma_2=0,\,\tau_{12}=\bar{\tau}_{12}\right)=1\end{aligned} \tag{18}$$

Then we write:

$$\sigma_1\left(\frac{1}{\bar{\sigma}_1^+}-\frac{1}{\bar{\sigma}_1^-}\right)+\sigma_2\left(\frac{1}{\bar{\sigma}_1^+}-\frac{1}{\bar{\sigma}_1^-}\right)+ \\ +\frac{\sigma_1^2}{\bar{\sigma}_1^+\bar{\sigma}_1^-}+\frac{\sigma_2^2}{\bar{\sigma}_2^+\bar{\sigma}_2^-}+\left(\frac{\tau_{12}}{\bar{\tau}_{12}}\right)^2=1 \tag{19}$$

According to experimental tests (Vasiliev *et al.* 2007), strength criterion (19) corresponds to experimental results better than criterion (15) and even more precisely than criteria (7) and (11).

Polynomial strength criteria show formal approximation of experimental data in the coordinates of principal axes, however. These criteria become more complex in other coordinates. Tensoric strength criteria are therefore applied. For example, when the orthotropic material moves from principal axes 1 and 2 to turned axes 1′ and 2′ at the angle $\phi=45^0$, the strength criterion is put in the following way:

$$F\left(\sigma_1,\sigma_2,\tau_{12}\right)=R_1\sigma_1+R_2\sigma_2+R_{11}\sigma_1^2+ \\ +R_{12}\sigma_1\sigma_2+R_{22}\sigma_2^2+S_{12}\tau_{12}^2=1 \tag{20}$$

When the marginal conditions are applied to receive constants, according to the equation (18) we receive:

$$F\left(\sigma_1,\sigma_2,\tau_{12}\right)=\left(\frac{1}{\bar{\sigma}_1^+}-\frac{1}{\bar{\sigma}_1^-}\right)\sigma_1+ \\ +\left(\frac{1}{\bar{\sigma}_2^+}-\frac{1}{\bar{\sigma}_2^-}\right)\sigma_2+\frac{\sigma_1^2}{\bar{\sigma}_1^+\bar{\sigma}_1^-}+ \\ +R_{12}\sigma_1\sigma_2+\frac{\sigma_2^2}{\bar{\sigma}_2^+\bar{\sigma}_2^-}+\left(\frac{\tau_{12}}{\bar{\tau}_{12}}\right)^2=1 \tag{21}$$

This criterion differs from the criterion (19) because new constant $R_{12}$ cannot be received, according to the conditions of equation (18).

Thus, the author suggests a tensoric criterion (Žiliukas 2006) , which is put in the following way:

$$m_1\sigma_i+m_2\sigma_0\le\sigma_{U\mu\sigma} \tag{22}$$

where $m_1,m_2$ – ultimate material's; $\sigma_{U\mu\sigma}$ – strength limit at $\mu_\sigma$ stress state; $\sigma_i$ – intensity of stresses (when $\sigma_x$ is used and $\tau_{xy}$ $\sigma_i=\frac{1}{\sqrt{2}}\sqrt{\sigma_x^2+3\tau_{xy}^2}$ ).

Average stress (when $\sigma_x$ is used and $\tau_{xy}$ $\sigma_0=\frac{\sigma_1+\sigma_2}{3}=\frac{\sigma_x}{3}$ ).

Parameter of stress state:

$$\mu_\sigma=\frac{2\sigma_2-\sigma_1-\sigma_3}{\sigma_1-\sigma_3}=\frac{2\sigma_2-\sigma_1}{\sigma_1}=-1$$

(at $\sigma_x$ and $\tau_{xy}$), i. e. $\sigma_1=\sigma_{u,t}$ while stretching, and while compressing when $\sigma_3$ stress is used, $\mu_\sigma=+1$ and $\sigma_3=\sigma_{u,c}$.

Then criterion (22) is put in the following way:

$$\frac{1}{\sqrt{2}}m_1\sqrt{\sigma_x^2+3\tau_{xy}^2}+m_2\frac{\sigma_x}{2}\le\sigma_{u,c} \tag{23}$$

When criterion (26) is written in non-linear form:

$$m_3\left(\sigma_x^2+3\tau_{xy}^2\right)+m_4\sigma_x^2\le\sigma_{u,c}^2\,, \tag{24}$$

we receive:

$$\left(m_3+m_4\right)\sigma_x^2+m_3\tau_{xy}^2\le\sigma_{u,c}^2\,. \tag{25}$$

## 4. Delamination analysis

In order to solve the delamination problem of composite, we should apply strength criterion (25). Taking into account equation (3), $\tau_{xy}=\frac{1}{2}\sin2\theta\sigma_x$, and (25), the strength criterion is put in the following way:

$$\sigma_x^2\left(m_3+m_4+\frac{1}{2}m_3\sin^2 2\theta\right)\le\sigma_{u,c}^2 \tag{26}$$

When the angle is $\theta=45^0$, we receive net shear and $\sigma_x=\frac{\sigma_{u,c}}{2}$, and when the angle is $\theta=0$, we receive axial compression and $\sigma_x=\sigma_{u,c}$. Then the constants $m_3$ and $m_4$ in equation (25) are calculated by these equations:

$$\begin{cases} m_3+m_4+\frac{1}{4}m_3=2\\ m_3+m_4=1\end{cases} \tag{27}$$

From where $m_3=4$; $m_4=-3$.

Thus, strength criterion (26) is put in the following way:

$$\sigma_x^2\left(1+\sin^2 2\theta\right)\le\sigma_{uc}^2 \quad (28)$$

or

$$\sigma_x \le \sqrt{\frac{\sigma_{u,c}^2}{1+\sin^2 2\theta}}. \quad (29)$$

Taking into account formula (9), and after we enter buckling force from formula (1) and do the operations, we receive:

$$L_{cr}^4=\frac{16\pi^4\left(EJ_{ef}\right)^2\left(1+\sin^2 2\theta_{cr}\right)\cos^2\theta}{A^2\left(V_r+nV_f\right)^2\sigma_{u,c}^2} \quad (30)$$

This formula determines the relation between the values of length $L_{cr}$ and shear angle $\theta_{cr}$ when a straight bar or plate made from composite is being buckled.

## 5. Experimental tests

In order to do experimental tests, a 12-mm-thick composite plate was chosen. It is laminated by $t_v=0.5\,\mathrm{mm}$ cover, the thickness of the resin is $t_m=2\,\mathrm{mm}$, and the thickness of the fiberglass is $t_f=7\,\mathrm{mm}$. This makes relative volume of filling $V_f=0.62$, and of matrix $V_r$ – 0.35. The modulus of elasticity are the following: filling– $E_f=45\,\mathrm{GPa}$, resin – $E_m=11\,\mathrm{GPa}$, and cover $E_v=E_m=11\,\mathrm{GPa}$. Thus the total modulus of elasticity received from the formula (2) is $E=30.89\,\mathrm{GPa}$. $E_f$ and $E_m$ proportion is $n=E_f/E_m=4.09$. According to ASTM D 638, the width of the sample is 12.7 mm.

Then the area of cross – section is $A=152.4\,\mathrm{mm}^2=152.4\cdot10^{-6}\,\mathrm{m}^2$.

Moment of inertia:

$$I_{ef}=I_{\min}=\frac{bh^3}{12}=\frac{12\cdot10^{-3}\left(12.7\cdot10^{-3}\right)^3}{12}=2.048\cdot10^{-9}\,\mathrm{m}^4$$

Strength limit of compression:

$$\sigma_{u,c}=3000\,\mathrm{MPa}=3\cdot10^9\,\frac{\mathrm{N}}{\mathrm{m}^2}\ \text{and}\ EI_{ef}=63\,\mathrm{N}\cdot\mathrm{m}^2$$

When we enter the values of experimental and calculated parameters into formula (30), we receive:

$$L_{cr}=1,28\sqrt[4]{1+\sin^2 2\theta_{cr}}\sqrt{\cos\theta_{cr}} \quad (31)$$

In such a way, if we have various values of critical delamination angle $\theta_{cr}$, we can calculate the critical length of the plate. The calculation results are presented in table.

**Table.** Dependencies of critical delamination angles and lengths of plate

| No | $\theta_{cr}$, degrees | $L_{cr}$, m |
|---|---|---|
| 1 | 0 | 1.28 |
| 2 | 5 | 1.286 |
| 3 | 10 | 1.308 |
| 4 | 28 | 1.3705 |
| 5 | 29 | 1.3706 |
| 6 | 30 | 1.39 |
| 7 | 31 | 1.368 |
| 8 | 32 | 1.367 |
| 9 | 40 | 1.103 |
| 10 | 45 | 1.076 |
| 11 | 90 | 0 |

According to table, the maximal critical length of the plates is received when the delamination angle is $30^0$.

## 6. Conclusions

1. The delamination of composite constructional elements is determined by the normal and shear stresses in the fiber.
2. The strength criteria used to evaluate the strength of composites are too complex because of the large number of constants and the difficulty of determining them.
3. The non-linear strength criterion suggested by the author for a complex state of stresses allows a simple dependency between critical delamination angles and critical lengths of the plate at buckling to be determined.
4. According to experimental and calculation data, the maximal critical length of the plate at buckling is received when the delamination angle is $30^0$.

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## KOMPOZITINIŲ KONSTRUKCIJŲ ELEMENTŲ ATSISLUOKSNIAVIMAS KLUPDANT

**A. Žiliukas**

S a n t r a u k a

Kompozitinių konstrukcijų elementų atsisluoksniavimas nustatomas remiantis normaliniais ir tangentiniais įtempiais. Autorius siūlo netiesinį stiprumo kriterijų, įvertinantį sudėtingą įtempių būvį sprendžiant kompozitų atsisluoksniavimo problemą.

**Reikšminiai žodžiai:** atsisluoksniavimas, kompozitas, stiprumo kriterijus.